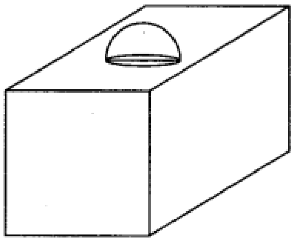
# Surface Areas and Volumes

## 2016

### Short Answer Type Questions II [3 Marks]

**Question 1.**

In Figure, a decorative block, made up of two solids a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block.(use π=22/7)



**Solution:**

Side of the cube = 6 cm

Total surface area of cube $= 6 \times (\text{side})^2 = 6 \times (6)^2 = 216 \text{ cm}^2$

Area covered on the face of cube by circular part of hemisphere $= \pi r^2 = \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \text{ cm}^2$

Curved surface area of hemisphere $= 2 \times \pi \times r^2 = 2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \text{ cm}^2$

So, Total surface area of the block = Surface area of cube – Area of circular face of hemisphere + Curved surface area of hemisphere

$$= 216 - \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} + 2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$$

$$= 216 + \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} = 216 + 9.625 = 225.625 \text{ cm}^2$$

**Question 2.**

A well of diameter 4 m is dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment

**Solution:**

Radius of the well = 2 m, height of the well = 21 m

Volume of the earth dug out $= \pi r^2 h$

$$= \pi \times 2 \times 2 \times 21 \text{ m}^3 = 264 \text{ m}^3$$

$\therefore$ Radius of embankment = Radius of well + width of embankment

$$= 2 + 3 = 5 \text{ m}$$

$$\text{Volume of embankment} = (\pi r_1^2 h - \pi r_2^2 h)\text{m}^3$$

$$= [\pi \times (5)^2 \times h - \pi(2)^2 h]\text{m}^3$$

$$= (\pi \times 25 \times h - \pi - 4h)\text{m}^3$$

$$= \left(\frac{22}{7} \times 21 \times h\right) \text{m}^3 = 66\,h \text{ m}^3$$

As per condition,

$\Rightarrow$ Volume of earth dug out = Volume of embankment

$$\Rightarrow \quad 264 \text{ m}^3 = 66h \text{ m}^3$$

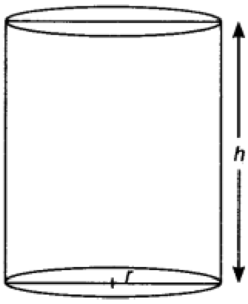
$$\Rightarrow \quad \text{Height of embankment, } h = \frac{264}{66} = 4 \text{ m}$$

**Question 3.**

The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder, (use π=22/7)

**Solution:**

Here $r + h = 37$ [Given, where $r \rightarrow$ radius, $h \rightarrow$ height]




$$\text{Total surface area of cylinder} = 2\pi r(h + r) = 2\pi rh + 2\pi r^2$$

$$\Rightarrow \quad 2\pi r(h + r) = 1628$$

$$\Rightarrow \quad 2\pi r \times 37 = 1628$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 37 = 1628$$

$$\Rightarrow \quad r = \frac{1628 \times 7}{2 \times 22 \times 37} = 7 \text{ cm}$$

Given, $r + h = 37$

$$7 + h = 37$$

$$\Rightarrow \quad h = 37 - 7 = 30 \text{ cm}$$

$$\text{Hence, volume of cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$$

**Question 4.**

A right circular cone of radius 3 cm, has a curved surface area of 47.1 cm2. Find the volume of the cone, (use π = 3.14)

**Solution:**

Radius of cone $(r)$ = 3 cm

Curved surface area $= \pi rl = 47.1 \text{ cm}^2$

$$\therefore \quad l = \frac{47.1}{\pi r} = \frac{47.1}{3.14 \times 3} = 5 \text{ cm, where } l = \text{slant height of cone}$$

We know that, $h = \sqrt{l^2 - r^2}$

$$= \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$$

$$\text{Volume of cone} = \frac{1}{3} \times \pi r^2 h$$

$$= \frac{1}{3} \times 3.14 \times 3 \times 3 \times 4 = 3.14 \times 3 \times 4 = 37.68 \text{ cm}^3$$

**Question 5.**

A toy is in the form of a cone of base radius 3.5 cm mounted on a hemisphere of base diameter 7 cm. If the total height of the toy is 15.5 cm, find the total surface area of the toy(use π =22/7)

**Solution:**

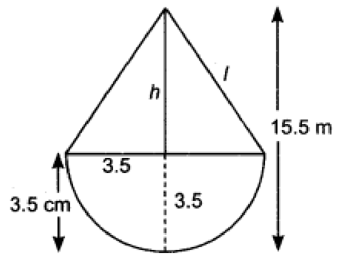
Here, given that $h = 15.5 - 3.5 = 12$ cm

Also, slant height of cone, $l = \sqrt{h^2 + r^2}$

$= \sqrt{(12)^2 + (3.5)^2}$ cm

$= \sqrt{144 + 12.25}$ cm

$= \sqrt{156.25}$ cm $= 12.5$ cm




$\therefore$ Curved Surface Area of cone $= \pi r l = \frac{22}{7} \times 3.5 \times 12.5$

$= 137.5 \text{ cm}^2$

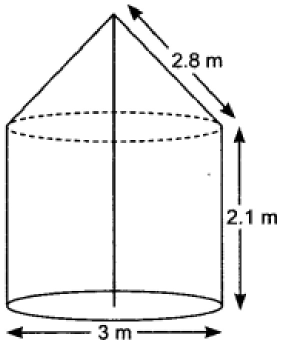
$\therefore$ Surface area of hemisphere $= 2\pi r^2 = 2 \times \frac{22}{7} \times (3.5)^2 = 77 \text{ cm}^2$

Hence, Total Surface Area of toy = Surface area of hemisphere + Curved Surface Area of cone

$= 77 + 137.5 = 241.5 \text{ cm}^2$

**Question 6.**

In figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at therate of rupees 500/sq. metre. (use π=22/7)




**Solution:**

Area canvas needed = curved surface area of cylinder + curved surface area of cone

$= 2\pi r h + \pi r l$ [where $r$ = radius, $h$ = height, $l$ = slant height]

$= 2 \times \frac{22}{7} \times 1.5 \times 2.1 + \frac{22}{7} \times 1.5 \times 2.8$

$= \frac{22}{7}[6.3 + 4.2] = \frac{22}{7} \times 10.5 = 33 \text{ m}^2$

Cost of canvas = 33 × 500 = ₹ 16500

**Question 7.**

A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel

**Solution:**

Volume of water in conical vessel

$= \frac{1}{3}\pi r^2 h$ [where $r$ = radius, $h$ = height]

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 24 \text{ cm}^3$$

Let height of water in cylindrical vessel be $h$ then volume of water in cylinder

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 10 \times 10 \times h \text{ cm}^3$$

A.T.Q.

Volume of water in conical vessel = Volume of water in cylindrical vessel

$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 24 = \frac{22}{7} \times 10 \times 10 \times h$$

$$\Rightarrow \quad h = \frac{5 \times 5 \times 24}{3 \times 10 \times 10} = 2 \text{ cm}$$

$\therefore$ Height to which water rises in cylindrical vessel = 2cm

**Question 8.**

A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by 3×5/9 cm. Find the diameter of the cylindrical vessel

**Solution:**

We know that, Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi \times (6)^3 = \frac{4}{3}\pi \times 216 \text{ cm}^3$$

Let radius of cylindrical vessel be $r$ cm then, volume of cylinder is:

$$V = \pi r^2 h = \pi r^2 \times \frac{32}{9} \text{ cm}^3$$

According to question,

volume of sphere = volume of cylinder

$$\frac{4}{3}\pi \times 216 = \pi \times r^2 \times \frac{32}{9}$$

$$r^2 = \frac{4 \times 216 \times 9}{3 \times 32} = 27 \times 3 = 81$$

$$r^2 = 81$$

$$r = 9 \text{ cm}$$

$\therefore$ Diameter of cylindrical vessel = $2r$ = 18 cm

**Question 9.**

A hemispherical tank, of diameter 3 m, is full of water. It is being emptied by a pipe 3×4/7 litre per second. How much time will it take to make the tank half empty?

**Solution:**

Radius of hemispherical tank = 1.5 m

Volume of water in hemispherical tank $= \frac{2}{3} \times \pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times (1.5)^3 = \frac{99}{14} \text{ m}^3$$

Rate of water taken out in 1 second $= \frac{25}{7}$ litre/second

Let time taken to empty to empty half the tank be '$t$' sec.

A.T.Q.,

$$\text{Rate of flow of water} \times t \text{ sec} = \frac{1}{2} \times \text{volume of water in the hemispherical tank}$$

$$\therefore \quad \frac{25}{7} \times \frac{1}{1000} \times t = \frac{1}{2} \times \frac{99}{14}$$

$$\therefore \quad t = 990 \text{ s}$$

$\therefore$ Time taken to empty half the tank = 16 min. 30 sec.

**Question 10.**

A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice-cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cone.

**Solution:**

$$\text{Volume of ice cream cylinder} = \pi r^2 h = \pi \times (6)^2 \times 15 \text{ cm}^3$$

$$\text{Volume of 1 ice cream cone} = \frac{1}{3} \times \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3} \times \pi \times r^2(4r) + \frac{2}{3}\pi r^3 \quad [\because \text{Height} = 2 \times \text{diameter}]$$

$$= \frac{4}{3}\pi r^3 + \frac{2}{3}\pi r^3 = \frac{6}{3}\pi r^3 = 2\pi r^3 \text{ cm}^3$$

$$\text{Volume of ice cream 10 such cones} = 10 \times 2\pi r^3 = 20\pi r^3 \text{ cm}^3$$

According to Question,

$$\text{Volume of 10 ice-cream cones} = \text{Volume of cylinder}$$

$$20\pi r^3 = \pi \times 36 \times 15$$

$$r^3 = \frac{36 \times 15}{20}$$

$$\Rightarrow \quad r^3 = 27 \Rightarrow r = 3 \text{ cm}$$

Diameter of conical ice cream cup = 6 cm

**Question 11.**

A metal container, open from the top, is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper circular ends are as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at of rupees 35/litre

**Solution:**

$$\text{Volume of milk in container} = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

=1/3 x 22/7 x 21[400+64+160]=22 x 624/1000 liters[1 $cm^3$=1/1000 l]

cost of 1l milk=35rupees

cost of milk in container=35  22 x 624/1000=480.48 rupees

## Long Answer Type Questions [4 Marks]

**Question 12.**

A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 $cm^3$. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket, (use π=3.14)

**Solution:**

Let R, r and V be the upper radius, lower radius and volume of the frustum respectively then R = 20 cm, r = 12 cm and V = 12508.8 $cm^3$

$$\text{Volume of frustum of cone} = \frac{1}{3} \times \pi[(R^2 + r^2 + Rr)]h$$

12308.8 = 1/3x 3.14 [400 + 144 +240]h

h=12308.8 x 3/3.14 x 784 h=15 cm

Now, l (slant height) =$\sqrt{(20-12)^2+15^2}=\sqrt{64+225}=\sqrt{289}=17$ $cm^2$

Total area of the metal sheet=curved surface area of the cone+area of the base=

π(R+r)l+π$r^2$

=π(20+12)x 17+π x 12 x 12

=π x 32 x 17 +144π =688π

=688 x 3.14 =2160.32 $cm^2$

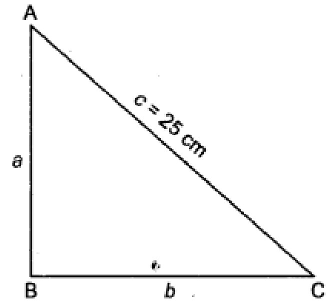
**Question 13.**

The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle

**Solution:**

Here, Perimeter of triangle, $a + b + c = 60$ [Given]

$$a + b + 25 = 60$$
$$a + b = 35 \text{ cm}$$




Using Pythagoras theorem,

$$a^2 + b^2 = (25)^2 = 625$$

Now, $$(a + b)^2 = a^2 + b^2 + 2ab$$
$$(35)^2 = 625 + 2ab$$
$$1225 - 625 = 2ab$$
$$\Rightarrow \quad 2ab = 600$$
$$\Rightarrow \quad ab = 300$$
$$\text{Area } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 300 = 150 \text{ cm}^2$$

**Question 14.**

Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ? 120 per sq.m, find the amount shared by each school to set up the tents.What value is genetated by above problem?

**Solution:**

$$\text{Slant height of conical part} = \sqrt{r^2 + h^2} = \sqrt{(2.8)^2 + (2.1)^2} = \sqrt{7.84 + 4.41}$$
$$= \sqrt{12.25} = 3.5 \text{ m}$$

Area of tents = Curved surface area of cylindrical part + Curved surface area of conical part

$$= 2\pi rh + \pi rl$$
$$= 2 \times \frac{22}{7} \times 2.8 \times 3.5 + \frac{22}{7} \times 2.8 \times 3.5$$
$$= 3 \times \frac{22}{7} \times 2.8 \times 3.5 = 92.4 \text{ m}^2$$

$\therefore$ Canvas required for 1500 tents $= 1500 \times 92.4 = 138600 \text{ m}^2$

Cost of 1500 tents $= (1500 \times 92.4) \times 120 =$ ₹ 16632000

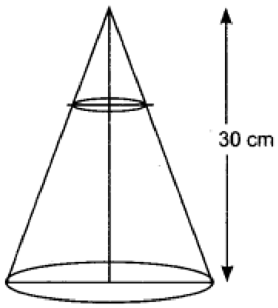
[$\because$ Making tents costs ₹ 120 per sq. m]

Share of each school $= \frac{1663200}{50} =$ ₹ 332640

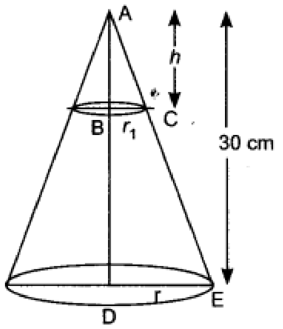
School authorities is concerned about safety of children and their families.

**Question 15.**

In figure, shown a right circular cone of height 30 cm. A small cone is cut off from the top by a plane parallel to the base. If the volume of the small cone is 1/27 of the volume of given cone, find at what height above the base is the section made.



**Solution:**




Let radius of big cone be $r_2 = r$ and small cone be $r_1$

In ΔADE and ΔABC we have

$$\frac{r_1}{r} = \frac{h}{30}$$

$$\Rightarrow \quad h = \frac{30r_1}{r}$$

Now, $$\frac{\text{Volume of small cone}}{\text{Volume of large cone}} = \frac{\frac{1}{3}\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h} = \frac{1}{27} \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{r_1^2 h}{r_2^2 \times 30} = \frac{1}{27}$$

$$\Rightarrow \quad \frac{r_1^2 \times 30 \times r_1}{r^2 \times 30 \times r} = \frac{1}{27}$$

$$\frac{r_1^3}{r^3} = \frac{1}{27} \Rightarrow \frac{r_1}{r} = \frac{1}{3}$$

$$\Rightarrow \quad h = 30 \times \frac{r_1}{r} = 30 \times \frac{1}{3} = 10$$

$$\therefore \quad h = 10 \text{ cm}$$

Thus, at a height 20 cm above base, a small cone is cut.

## 2015

## Short Answer Type Questions II [3 Marks]

**Question 16.**

In fig, from the top of a solid cone of a height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid

**Solution:**

The plane PQ cuts the solid cone such that PQCB becomes a frustum.

Let radius of top of frustum be $r_1$.

Radius of bottom of frustum, $r_2 = 6$ cm.

Height of frustum, $h = 12 - 4 = 8$ cm.

Now, in ΔAMQ & ΔANC

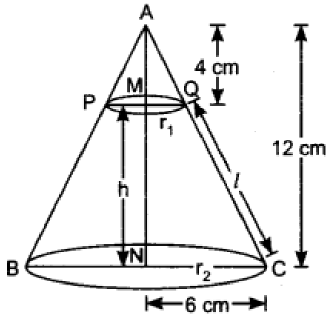
$\angle M = \angle N$ [Corresponding angles]

$\angle Q = \angle C$ [Corresponding angles]

So, ΔAMQ ~ ΔANC [By AA similarity criteriron]

∴ ΔAMQ ~ ΔANC

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{AM}{AN} \Rightarrow \frac{r_1}{6} = \frac{4}{12} \Rightarrow r_1 = 2 \text{ cm}$$




Now, slant height of frustum, $l = \sqrt{(r_2 - r_1)^2 + h^2}$

$$= \sqrt{(6-2)^2 + 8^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5} \text{ cm.}$$

Now, Total surface area of frustum $= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

$$= \pi(2+6)4\sqrt{5} + \pi \times (2)^2 + \pi \times (6)^2$$

$$= \pi(32\sqrt{5} + 4 + 36) = 350.59 \text{ cm}^2$$

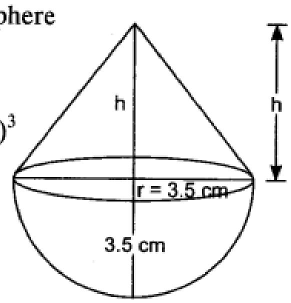
**Question 17.**

A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in making of toy is 166 x 5/6 cm3 Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of 10 rupees per cm2

**Solution:**

**Hemisphere:** Radius of hemisphere, $r = 3.5$ cm

**Cone:** Radius of cone, $r = 3.5$ cm and let height of cone be '$h$'. Then,




$$\text{Volume of toy} = \text{Volume of cone} + \text{Volume of hemisphere}$$

$$166\frac{5}{6} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$\Rightarrow \quad 166\frac{5}{6} = \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 h + \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$$

$$\Rightarrow \quad \frac{1001}{6} = \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 (h + 2 \times 3.5)$$

$$\Rightarrow \quad \frac{1001}{6} = \frac{22 \times 3.5 \times 3.5}{3 \times 7}(h + 7)$$

$$\Rightarrow \quad (h + 7) = \frac{1001 \times 3 \times 7}{22 \times 3.5 \times 3.5 \times 6} = 13 \quad \Rightarrow \quad h = 13 - 7 = 6 \text{ cm}$$

$\therefore$ Height of cone, $h = 6$ cm

$\therefore$ Height of toy $= h + r = 6 + 3.5 = 9.5$ cm

Curved surface area of hemisphere $= 2\pi r^2$

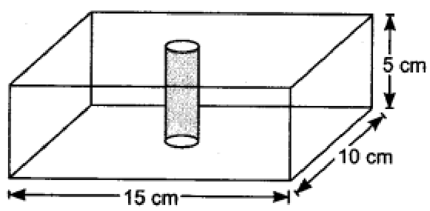
$$= 2 \times \frac{22}{7} \times (3.5)^2 = 77 \text{ cm}^2$$

Cost of painting hemispherical part of toy = ₹ 10 per cm$^2$

$\therefore$ Total cost of painting hemispherical portion of toy = ₹ $(10 \times 77)$ = ₹ 770.

**Question 18.**

In figure, from a cuboidal solid metallic block of dimensions 15 cm X 10 cm X 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block.




**Solution:**

Cuboid: Length of cuboid, l = 15 cm

Breadth cuboid, b = 10 cm

Height of cuboid, h = 5 cm

Cylinder: Diameter of cylinder = 7 cm

Radius of cylinder, r = 7/2 cm

Height of cylinder, h' = 5 cm

Surface area of remaining block = Total surface area of cuboidal block + Curved Surface Area of cylinder – Area of 2 circles

$$= 2(lb + bh + hl) + 2\pi r h' - 2(\pi r^2)$$

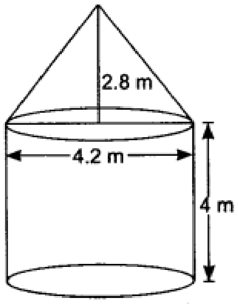
= 2(15 x 10 + 10 x 5 + 5 x 15) + 2 x 22/7 x 7/2 x 5- 2 x 22/7 x 7/2 x 7/2

= 2(150 + 50 + 75) + 110 – 77 = 550 + 33 = 583 cm².

**Question 19.**

Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but of height 2.8 m, and the canvas to be used costs 100 rupees per sq. m, find the amount, the associations will have to pay.What values are shown by these associations ?

**Solution:**




Height of cylinder = 4 m

Radius =2.1 m

Curved surface area of cylinder = $2\pi rh = 2 \times 22/7 \times 2.1 \times 4 = 52.8\ m^2$

Radius of cone = 2.1 m, height of cone = 2.8 m.

Let slant height = l

$l = \sqrt{r^2+h^2}$ [ Pythagoras theorem]

Slant height of cone, $l = \sqrt{(2.1)^2 + (2.8)^2} = 3.5$ m

Curved surface area of cone $=\pi rl = 22/7 \times 2.1 \times 3.5 = 23.1\ m^2$

Area of canvas required for one tent = $52.8 + 23.1 = 75.9\ m^2$

Canvas required for 100 tents = $100 \times 75.9 = 7590\ m^2$

Total cost = rupees (7590 x 100) = rupees 759000

Amount to be paid by association = 50/100 x 759000 = 379500 rupees

Care for the society, values shown by welfare associations.

**Question 20.**

A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer.

**Solution:**

Radius of hemispherical bowl = 18 cm

Volume of liquid in bowl = $2/3\pi r^3$

$=2/3 \times \pi \times 18 \times 18 \times 18 = 3888\pi\ cm^3$

liquid wasted=$10/100 \times 3888\pi\ cm^3 = 3888\pi/10\ cm^3$

liquid transferred into bottles =$3888\pi - 3888\pi/10$

$=34992\pi/10\ cm^3$

radius of bottle=3cm

let height = X cm

volume of bottle $=\pi r^2 h$

$=\pi \times 3 \times 3 \times X = 9\pi X\ cm^3$

volume of 72 bottles $=72 \times 9\pi X\ cm^3$

$=648\pi X\ cm^3$

$648\pi X = 34992\pi/10$

X=34992/10 x 648 =5.4 cm

Height of each bottle = 5.4 cm

**Question 21.**

A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have? Find the cost of painting the total surface area of the solid so formed, at the rate of 5 rupee per 100 sq. cm

**Solution:**

Largest diameter of hemisphere = 10 cm = Side of cube

.'. radius = 5 cm

Total surface area of the solid = Surface area of cube +Curved surface area of hemisphere – Area of base of hemisphere

$= 6(\text{side})^2 + 2\pi r^2 - \pi r^2$

$= 6 \times 10^2 + 2 \times 3.14 \times 5 \times 5 - 3.14 \times 5 \times 5$

$= 600 + 78.5 = 678.5\ cm^2$

Total cost = 678.5 x 5 /100 =

33.92 rupees

**Question 22.**

504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area

**Solution:**

Volume of the cone $=1/3 \times \pi r^2 h$

$=1/3\pi \times 3.5/2 \times 3.5/2 \times 3\ cm^3 = 12.25/4\ \pi cm^3$

volume of 504 $=504 \times 12.25/4\ \pi cm^3 = 1543.5\ \pi cm^3$

volume of the sphere=$1543.5\ \pi^3$=volume of 504 cones

$4/3\pi r^3=1543.5\pi$

$r^3 1543.5\times3/4=1157.625$

$r=3\sqrt{1157.625}=10.5cm$

Diameter of sphere=21cm

surface area of sphere=$4\pi r^2$

$=4\times22/7\times10.5\times10.5=1386cm^2$

**Question 23.**

Two spheres of same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere.

**Solution:**

Radius of smaller sphere = 3 cm

Volume of smaller sphere $=4/3\pi \text{x}3\text{x}3\text{x}3cm^3=36\pi\ cm^3$

When Mass=1kg,then volume=$36\pi\ cm^3$

When Mass = 7 kg

volume=$7\times36\pi\ cm^3=252\pi\ cm^3$

total value of two spheres=$36\pi+252\pi = 288\pi\ cm^3$

let the radius of sphere so formed=R cm

volume of big sphere=total volume of 2 sphere

$4/3\pi R^3=288\pi$

$R^3=288 \times 3/4=72 \times 3 =216$

R=6 cm

Diameter =12 cm

**Question 24.**

A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weight, a conical hole is drilled in the cylinder. The conical hole has a radius of 3/2 cm and its depth 8/9 cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape

**Solution:**

Given, Radius of the cylinder = 3 cm

Height = 5 cm

Volume of cylinder = $\pi r^2 h = \pi \times 3 \times 3 \times 5\ cm^3 = 45\pi\ cm^3$

Radius of cone = 3/2 cm

Height of the cone =8/9 cm

Volume of the cone $=1/3\ \pi r^2h$

$=1/3\pi \times 9/4 \times 8/9 = 2\pi/3 cm^3$

Volume of metal left $=45\pi-2\pi/3=133\pi/3\ cm^3$

Ratio=volume of metal left in cylinder/volume of the metal taken out=$133\pi/3:2\pi/3=133:2$

**Question 25.**

A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right- circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder, in cubic metres

**Solution:**

Radius of cone = 30 cm

height of cone = 60 cm

Volume of cone $= 1/3\pi r^2 h = 1/3\pi \times 30 \times 30 \times 60 = 18000\ \pi\ cm^3$

Radius of cylinder = 60 cm

Height of cylinder = 180 cm

Volume of cylinder $=\pi r^2h$

$=\pi \times 60 \times 60 \times 180\ cm^3 = 648000\ \pi\ cm^3$

volume of water left=volume of cylinder-volume of cone

$=648000\pi-18000\pi$

$=630000\ \pi\ cm^3$

$=630000 \times 22/7 = 1980000\ cm^3 = 1.98\ m^3$

**Question 26.**

The rain water from a 22 m x 20 m roof drains into a cylindrical vessel of diameter 2 m and height 3.5 m. If the rain water collected from the roof fills 4/5th of the cylindrical vessel, then find the rainfall in cm

**Solution:**

Let the rainfall=x m

volume of rain water=lbh

$=22 \times 20 \times X = 440X\ m^3$

radius of cylindrical vessel=1 m

height =3.5m

volume of vessel$=\pi r^2h$

$=22/7 \times 1^2 \times 3.5=22 \times 0.5 = 11\ m^3$

now,it is given that vessel is filled upto 4/5 of the volume by rain water

$4/5 \times 11=440X$

$4\times11/5\times440=X$

$X=1/50$

$X=1/50\times100=2$ cm

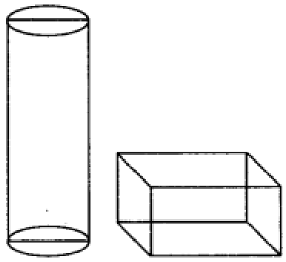
rainfall=2cm

## Long Answer Type Questions [4 Marks]

**Question 27.**

A 21 m deep well with diameter 6 m is dug and the earth from digging is evenly spread to form a platform 27 m x 11 m. Find the height of the platform.

**Solution:**



cylinder:Diameter of cylinder =6 m

radius of cylinder,r=6/2=3 m

height of cylinder,h=21 m

cuboid of platform:Lenght=21m,breadth=11m,height=h

according to question,

volume of cuboid platform=volume of cylindrical well

$lbh'=\pi r^2 h$

27x11xh'=22/7x3x3x21

h'=22x3x3x21/7x27x11=2m

height of platform =2m

**Question 28.**

A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the width of the embankment.

**Solution:**

Given, Diameter of the well = 4 m

Radius of the well = 2 m

Depth of the well = 14 m = Height

Volume of earth taken out from the well = $\pi r^2 h$

= πx2x2x14 = 56π $m^3$

Earth taken out from the well evenly spread to form an embankment having height 40 cm = 0.4 m

Let external radius of embankment be R.

Internal radius = 2 m = radius of well

Volume of embankment (cylindrical)=$\pi(R^2-r^2)h$

=$\pi(R^2-4)$ x 4/10 =56π

$R^2$-4=56×10/4=140

$R^2$=140+4=144

R=12m

Width of embankment=R-r=12-2=10m

**Question 29.**

Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm, If the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe

**Solution:**

Let the radius of the pipe=r cm

Speed of water=2.52 km/hr=2520 m/h

Volume of the water that flows in half an hour=$1/2\pi r^2 h$

=1/2π x r/100 x r/100 x 2520 =$126\pi r^2/1000$ $m^3$

volume of the water in cylindrical tank=π x 40/100 x 40/100 x 3.15 $m^3$

volume of the water flowing in 1/2hr=volume of water in cylindrical tank

$126\pi r^2/1000$=π x 40/100 x 40/100 x 3.15

$r^2$=40/100 x 40/100 x 3.15 x 1000/126

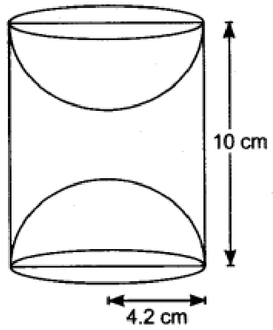
$r^2=4 \Rightarrow r=2$ cm

internal diameter of pipe=4cm

**Question 30.**

From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire

**Solution:**




Given, Radius of hemisphere=4.2cm

Volume of hemisphere=$2/3\pi r^3$=$2/3\pi$ x $(4.2)^3$ $cm^3$=$49.392\pi$ $cm^3$

Volume of 2 hemispheres =π x $49.392\pi$ $cm^3$

=$98.784\pi$ $cm^3$

Height of cylinder =10cm

Radius =4.2cm

Volume of cylinder =$\pi r^2 h$

=π x $(4.2)^2$ x 10=176.4π

Volume of metal left =176.4π-98.784π

=$77.616\pi$ $cm^3$

Radius of wire =0.7 cm

Let length of wire =X cm

Volume of cylindrical wire=π x 0.7 x 0.7 x X

=$0.49\pi X$ $cm^3$

Volume of cylindrical wire =volume of metal left from cylinder

0.49πX=77.616π

X=77.616/0.49=158.4 cm

Length of wire=158.4 cm

**Question 31.**

A vessel full of water is in the form of an inverted cone of height 8 cm and the radius of its top, which is open, is 5 cm. 100 spherical lead balls are dropped into the vessel. One-fourth of the water flows out of the vessel. Find the radius of a spherical ball.

**Solution:**

Radius of cone = 5 cm and height of cone = 8 cm

.'. Volume of the cone=$1/3\pi r^2 h$

=1/3π x 5 x 5 x 8 =200/3π $cm^3$

Let radius of spherical lead ball =r cm

Volume of lead ball=$4/3\pi r^3$ $cm^3$

A.T.Q., volume of 100 lead balls=1/4 x volume of cone

100 x $4/3\pi r^3$=1/4 x 200/3π

$400r^3$=50

$r^3$=50/400=1/8

r=1/2=0.5 cm

Radius of spherical lead ball=0.5 cm

**Question 32.**

Milk in a container, which is in the form of a frustum of a cone of height 30 cm and the radii of whose lower and upper circular ends are 20 cm and 40 cm respectively, is to be distributed in a camp for flood victims. If this milk is available at the rate of 35 rupees per litre and 880 litres of milk is needed daily for a camp, find how many such containers of milk are needed for a camp and what cost will it put on the donor agency for this. What value is indicated through this by the donor agency

**Solution:**

Given, $R_1$ = 20 cm, $R_2$ = 40 cm, height = 30 cm, where $R_1$ = radius of lower end,

$R_2$ = radius of bigger end at top

Volume of the container open at top$=1/3\pi h[R_1^2+R_2^2+R_1R_2]$

$=1/3\pi \times 30(20^2+40^2+20 \times 40)$

$=28000\pi=28000 \times 22/7= 88000$ cm³

Now, 880 liters=880 x 1000 cm³

=880000 cm³

Number of container of milk=880000/88000=10

Cost of 880 l of milk= 880 x 35 =30800 rupees

The moral value depicted is kindness

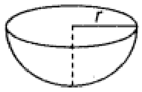
## 2014

### Very Short Answer Type Question [1 Mark]

**Question 33.**

If the total surface area of a solid hemisphere is 462 cm², find its volume.

**Solution:**




Total surface area of solid hemisphere$=2\pi r^2+\pi r^2=3\pi r^2$

Now given $3\pi r^2=462$

$r^2=462 \times 7/3 \times 22 =49$

r=7cm

volume of hemisphere$=2/3\pi r^3=2/3 \times 22/7 \times (7)^3=2/3 \times 22/3 \times 22 \times 49$ cm³

=718.67 cm³

### Short Answer Type Questions II [3 Marks]

**Question 34.**

A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 4 km per hour, in how much time will the tank be filled competely

**Solution:**

Internal diameter of pipe = 20 cm

So,internal radius of pipe=10cm=1/10m

In 1 hour, 4 km = 4000 m length of water which flows in the tank.

So, volume of water in 1 hour which flows out $=\pi r^2h=\pi(1/10)^2 \times 4000=40\pi m^3$

Diameter of cylindrical tank = 10 m

So, radius of cylindrical tank = 5m ,

Height of cylindrical tank = 2 m

.'. Volume of cylinder tank $=\pi r^2h=\pi \times 5 \times 5 \times 2 =50\pi$ m³

Time taken to fill the tank=volume of the tank/volume of the water in 1hr

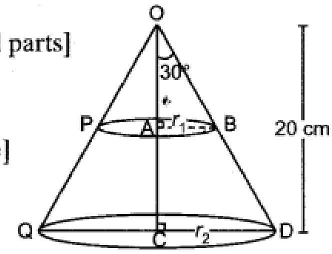
=50π/40π=5/4 hrs=1 hr 15 minutes

**Question 35.**

A solid metallic right circular cone 20 cm high and whose vertical angle is 60°, is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter 1/12 cm, find the length of wire

**Solution:**

Cone is cut by plane PB and PQDB is a frustum.




$$OA = AC = 10 \text{ cm}, \ [\because \text{ equal parts}]$$
$$AB = r_1;\ CD = r_2$$
$$\text{Vertical angle} = 60°$$
So, $$\text{semi-vertical angle} = 30° \ [\because \text{ ASP of triangle}]$$
In right angled ΔOAB,
$$\frac{AB}{OA} = \tan 30°$$
$$\Rightarrow \frac{r_1}{10} = \frac{1}{\sqrt{3}} \Rightarrow r_1 = \frac{10}{\sqrt{3}} \text{ cm}$$

In right angled ΔOCD,
$$\frac{CD}{OC} = \tan 30° \Rightarrow \frac{r_2}{20} = \frac{1}{\sqrt{3}} \Rightarrow r_2 = \frac{20}{\sqrt{3}} \text{ cm}$$

Frustum is drawn into wire of diameter $\frac{1}{12}$ cm and length $x$ cm (say).

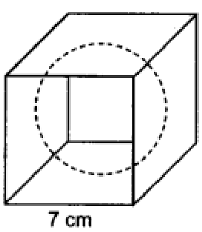
∴ A.T.Q., Volume of cylindrical wire = Volume of frustum
$$\pi r^2 x = \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$
$$\pi\left(\frac{1}{24}\right)^2 x = \frac{1}{3}\pi \times 10\left[\left(\frac{10}{\sqrt{3}}\right)^2 + \left(\frac{20}{\sqrt{3}}\right)^2 + \frac{10}{\sqrt{3}}\cdot\frac{20}{\sqrt{3}}\right]$$
$$\frac{x}{576} = \frac{10}{3}\left[\frac{100}{3} + \frac{400}{3} + \frac{200}{3}\right]$$
$$\Rightarrow \frac{x}{576} = \frac{10}{3} \times \frac{700}{3} \Rightarrow x = \frac{7000}{9} \times 576$$
$$\Rightarrow x = 7000 \times 64 \text{ cm}$$
$$\Rightarrow x = 448000 \text{ cm}$$
$$\therefore \text{Length of wire} = 448000 \text{ cm} = 4.48 \text{ km}$$

**Question 36.**

The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left

**Solution:**

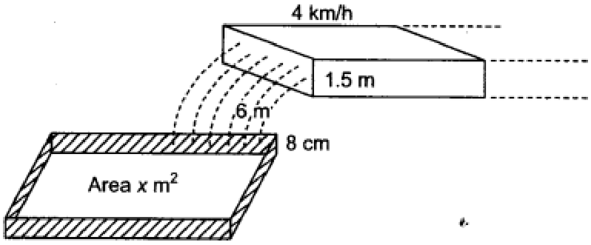
As largest possible sphere is carved out of a solid wooden cube of side 7 cm, so, diameter of the sphere is 7 cm.




∴ Volume of the wood left = Volume of the cube – Volume of the sphere
$$= (\text{side})^3 - \frac{4}{3}\pi r^3$$
$$= \left[(7)^3 - \frac{4}{3}\pi\left(\frac{7}{2}\right)^3\right] \text{cm}^3 = (7)^3\left[1 - \frac{4}{3} \times \frac{22}{7} \times \frac{1}{8}\right]$$
$$= 343[1 - 0.52] = 343 \times 0.48 \text{ cm}^3 = 164.64 \text{ cm}^3.$$

**Question 37.**

Water in a canal, 6 m wide and 1.5 m deep, is flowing at a speed of 4 km/h. How much area will it irrigate in 10 minutes, if 8 cm of standing water is needed for irrigation?




**Solution:**

Given; canal is 6 m wide, 1.5 m deep

and in 1 hour, 4 km length of water flows out.

.'. Volume of water flows out in 1 hour = l x b x h = 6 x 1.5 x 4000 $m^3$ = 36000 $m^3$

.'. Volume of water flows out in 10 minutes=36000/60 x 10 = 6000 $m^3$—(1)

Suppose this water irrigates X $m^2$ of area and we require 8 cm of standing water.

.'. Volume of water required = Area of cross-section x Length= X x 8/100 $m^3$—-(2)

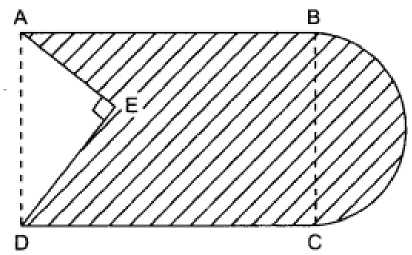
from 1& 2,we get

X x 8/100 =6000⇒X=6000 x 100/8 =75000 $m^2$

∴75000 $m^2$ of area is irrigated

**Question 38.**

In Figure, from a rectangular region ABCD with AB = 20 cm, a right triangle AED with AE = 9 cm and DE = 12 cm, is cut off. On the other end, taking BC as diameter; a semicircle is added on outside the region. Find the area of the shaded region



[Pythagoras theorem]

**Solution:**

Triangle AED is right-angled at E.

$\therefore AD^2=AE^2+ED^2$

$AD=\sqrt{9^2+12^2}=\sqrt{81+144}=\sqrt{225}=15$ cm

BC=AD=15 cm

Now, Area of shaded portion = Area of rectangle + Area of Semicircle – Area of triangle AED

= l x b x $1/2\pi r^2$-1/2 x base x height

=[20 x 15 +$1/2\pi(15/2)^2$-1/2 x 9 x 12] $cm^2$

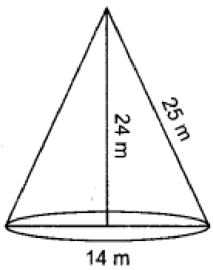
=[300+1/2 x 3.14 x $(15/2)^2$-54] $cm^2$

=[246 + 88.31] $cm^2$= 334.31 $cm^2$

**Question 39.**

A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m.Find the cost of cloth used at the rate 25 rupees /metre.

**Solution:**




Base diameter of conical tent = 14 m and height = 24 m.

∴Slant height(l)=$\sqrt{(7)^2+(24)^2}$ m

=$\sqrt{49+576}$ m =$\sqrt{625}$ m =25 m

Area of the cloth required for conical tent=πrl =22/7 x 7 x 25 =550 $m^2$

Width of cloth=5 m

Length of cloth required =550/5 =110 m

Cost of cloth = 25 rupee x 110 =2750 rupees

**Question 40.**

A girl empties a cylindrical bucket, full of sand, of base radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap 24 cm, then find its slant height correct upto one place of decimal

**Solution:**

For cylindrical bucket:

Base radius = 18 cm and height = 32 cm.

For cone: Height = 24 cm.

Let base radius = r cm

When cylinder is converted into cone, then their volumes are equal.

.'. Volume of cone = Volume of cylinder

=$1/3\pi r^2 \times 24 = \pi(18)^2 \times 32$

=> $r^2 = (18)^2 \times 4$

r = 18 x 2 = 36 cm

Radius of the base of cone = 36 cm

Slant height (l) =$\sqrt{(24)^2 + (36)^2}$cm = $\sqrt{576+1296}$ cm

[∴Using Pythagoras theorem $l = \sqrt{h^2 + r^2}$]

= $\sqrt{1872}$ cm = 43.26 cm = 43.3 cm

## Long Answer Type Questions [4 Marks]

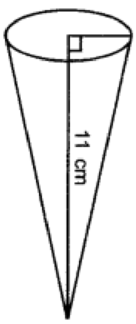
**Question 41.**

Sushant has a vessel, of the form of an inverted cone, open at the top, of height 11 cm and radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put in the vessel due to which 2/5 th of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the flower beds. What value has been shown by Sushant

**Solution:**

Radius of base of the cone = 2.5 cm and

height = 11 cm




∴ Volume of water in cone = $\frac{1}{3}\pi(2.5)^2 \times 11\ cm^3$

$\frac{2}{5}$th of the volume of water in cone = $\frac{2}{5} \times \frac{\pi}{3} \times (2.5)^2 \times 11\ cm^3$

Diameter of spherical ball = 0.5 cm

Volume of spherical ball = $\frac{4}{3}\pi\left(\frac{0.5}{2}\right)^3$

Let '$n$' spherical balls be dropped.

We know volume of water displaced is equal to volume of body immersed.

$$\therefore \quad n \times \frac{4}{3}\pi\left(\frac{0.5}{2}\right)^3 = \frac{2}{5} \times \frac{\pi}{3} \times (2.5)^2 \times 11$$

$$n = \frac{2}{5} \times (2.5)^2 \times 11 \times \frac{1}{(0.5)^3} \times \frac{8}{4}$$

$$\therefore \quad n = \frac{4 \times 2.5 \times 2.5 \times 11}{5 \times 0.5 \times 0.5 \times 0.5} = 440$$

∴ 440 spherical balls were put in the vessel.

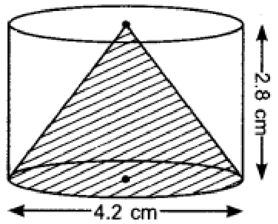
The moral value depicted is to save every drop of water.

**Question 42.**

From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid

**Solution:**

The shaded conical cavity is hollowed out.




**For cylinder:**

Radius of base, $r = \frac{4.2}{2} = 2.1$ cm

height, $h = 2.8$ cm

**For cone:** Radius of base, $r = \frac{4.2}{2} = 2.1$ cm

height, $h = 2.8$ cm

slant height, $l = \sqrt{(2.1)^2 + (2.8)^2} = \sqrt{4.41 + 7.84}$ $[\because \; l^2 = r^2 + h^2]$

$= \sqrt{12.25} = 3.5$ cm.

Now, Total surface area of remaining solid

= Curved surface area of cylinder + Area of top circular base + Curved surface area of cone

$= 2\pi rh + \pi r^2 + \pi rl = \pi r[2h + r + l]$

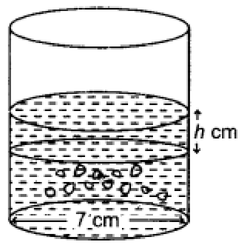
$= \pi \times (2.1)[2 \times 2.8 + 2.1 + 3.5]\ \text{cm}^2$

$= \frac{22}{7} \times 2.1 \times (5.6 + 5.6) = \frac{22}{7} \times 2.1 \times 11.2\ \text{cm}^2 = 73.92\ \text{cm}^2$

**Question 43.**

150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

**Solution:**




Let rise in water level in cylinder be h cm when 150 spherical marbles,

each of diameter 1.4 cm are dropped and fully immersed.

Then volume of 150 spherical marbles=volume of water raised in cylinder

$150 \times 4/3\pi(0.7)^3 = \pi(7/2)^2 h$

150 x 4/3 x 7 x 7 x 7/1000 =7 x 7/4 x h

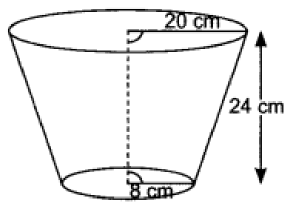
h= 4 x 4 x 7/20=28/5 =5.6 cm

water level rise by 5.6 cm

**Question 44.**

A container open at the top, is in the form of a frustum of a cone of height 24 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of rupee 21 per litre. 22

**Solution:**




Radius of lower end $(r_1)$ = 8 cm Radius of upper end $(r_2)$ = 20 cm

Height of frustum = 24 cm

Volume of the container, in form of frustum of cone,

$=\pi h/3[r1^2+r2^2+ r1r2]$

$= 22/7 \times 24/3[(8)^2 + (20)^2 + 8 \times 20]$

$=22/7 \times 8 [64 + 400 + 160] =22/7 \times 8 \times 624\ cm^3= 15689.14\ cm^3$

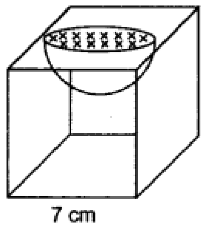
= 15.68914 L

.’. Cost of milk which can completely fill the container = 21 x 15.68914 = 329.47 rupees.

### Question 45.

A hemispherical depression is cut out from one face of a cubical block of side 7 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of remaining solid

**Solution:**




Edge of the cube=7 cm

Diameter of hemisphere =7 cm

Now,surface area of remaining solid=surface area of 6 faces of cube+surface area of hemisphere-surface area of circular top of diameter 7 cm

$6(edge)^2+2\pi r^2-\pi r^2$

$=6(7)^2+2\pi r(7/2)^2-\pi(7/2)^2$

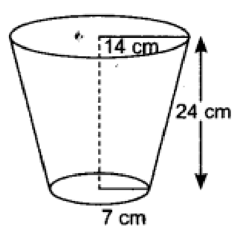
$=[6 \times 49+22/7 \times 49/4]\ cm^2 =[294 +38.5]cm^2 =332.5\ cm^2$

### Question 46.

A metallic bucket, open at the top, of height 24 cm is in the form of the frustum of a cone, the radii of whose lower and upper circular ends are 7 cm and 14 cm respectively. Find:

1. the volume of water which can completely fill the bucket
2. the area of the metal sheet used to make the bucket.

**Solution:**




For bucket,

radius of lower end $(r_1)$ = 7 cm,

radius of upper end $(r_2)$ = 14 cm

and height = 24 cm.

(*i*) Volume of water which can completely fill the bucket

$$= \frac{4}{3}h[r_1^2 + r_2^2 + r_1r_2]$$

$$= \frac{\pi}{3} \times 24\,[(7)^2 + (14)^2 + 7 \times 14]\ cm^3$$

$$= 8 \times \frac{22}{7}\,[49 + 196 + 98]\ cm^3 = 8 \times \frac{22}{7} \times 343\ cm^3 = 8624\ cm^3$$

(*ii*) Slant height ($l$) of bucket $= \sqrt{(24)^2 + (14-7)^2}$ cm $\quad [\because\ l^2 = h^2 + r^2]$

$$= \sqrt{576+49}\ cm = \sqrt{625}\ cm = 25\ cm$$

$\therefore$ Area of metal sheet used = Curved surface area of cone + Area of circular base

$$= \pi l(r_1 + r_2) + \pi r_1^2$$

$$= [\pi \times 25(7 + 14) + \pi(7)^2]\ cm^2$$

$$= [\pi \times 25 \times 21 + \pi \times 49]\ cm^2 = 7\pi[75 + 7]$$

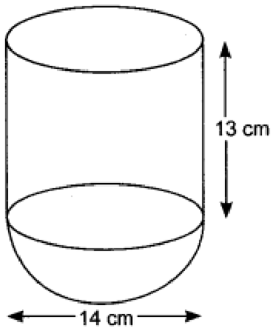
$$= 7 \times \frac{22}{7} \times 82\ cm^2 = 1804\ cm^2$$

## Short Answer Type Questions II [3 Marks]

**Question 47.**

A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter, the diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel.

**Solution:**




Diameter of hemispherical bowl = 14 cm

Radius of hemispherical bowl = 7 cm

Height of cylinder = 13 cm – 7 cm = 6 cm

Radius of cylinder = 7 cm Now,

Total surface area of vessel

= Curved Surface Area of hemispherical bowl +

curved surface area of cylinder

$= 2\pi r^2 + 2\pi rh = 2\pi \times 7 \times 7 + 2\pi \times 7 \times 6$

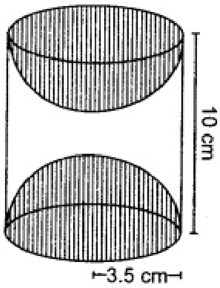
$= 98\pi + 84\pi = 182\pi$ cm²

$= 26 \times 22 = 572$ cm²

**Question 48.**

A wooden toy was made by scooping out a hemispherical of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy.

**Solution:**




Radius of cylinder r = 3.5 cm ; Height of cylinder h = 10 cm Radius of hemisphere = 3.5 cm

Remaining volume of wood in toy

= Volume of cylinder – Volume of two hemispherical scoops

$= \pi r^2 h - 2 \times 2/3 \pi r^3$

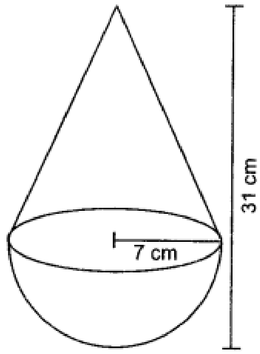
$= \pi \times (3.5)^2 \times [10 - 4/3 \times 3.5]$

$= 22/7 \times 35/10 \times 35/10 \times 16/3 = 1232/6 = 205.33$ cm³

**Question 49.**

A toy is in the form of a cone mounted on a hemisphere of same radius 7 cm. If the total height of the toy is 31 cm, find its total surface area.

**Solution:**




Radius of hemisphere = height of hemisphere = 7cm

Height of cone = total height – (height of hemisphere)

= 31 – 7 = 24 cm

Slant height of cone =l $=\sqrt{h^2+r^2}=\sqrt{(24)^2(7)^2}$

$=\sqrt{576+49}=\sqrt{625}=25$ cm

Curved surface area of cone = πrl =22/7 x 7 x 25 = 550 $cm^2$

Curved surface area of hemisphere = $2\pi r^2$

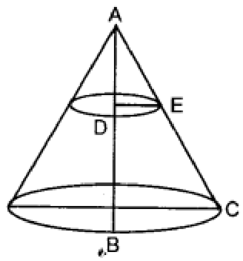
= 2 x 22/7 x 7 x 7 = 308 $cm^2$

Total surface area = 550 + 308 = 858 $cm^2$

**Question 50.**

A solid cone of base radius 10 cm is cut into two parts through the mid-point of its height, by a plane parallel to its base. Find the ratio of the volumes of the two parts of the cone

**Solution:**




Let height of cone = H = $2x$ cm

$\Rightarrow$ AB = $2x$ cm and AD = $x$ cm

R = radius of cone = BC = 10 cm

Let radius of smaller cone = $r$ cm. Clearly,

$$\Delta ABC \sim \Delta ADE \quad [\because \text{By AA smilarity}]$$

$\Rightarrow$ ∴ By c.p.s.t. $\dfrac{AB}{AD} = \dfrac{BC}{DE}$

$\Rightarrow$ $\dfrac{2x}{x} = \dfrac{10}{r} \Rightarrow r = 5$ cm

$$\text{Volume of bigger cone} = \frac{1}{3}\pi R^2 H = \frac{1}{3}\pi(10)^2 \times 2x = \frac{200}{3}\pi x \text{ cm}^3.$$

$$\text{Volume of smaller cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5)^2 \times x = \frac{25}{3}\pi x \text{ cm}^3$$

Volume of lower part, i.e. frustum = Volume of bigger cone – Volume of smaller cone

$$= \frac{200}{3}\pi x - \frac{25}{3}\pi x = \frac{175}{3}\pi x \text{ cm}^3$$

Now, $$\frac{\text{Volume of smaller cone}}{\text{Volume of lower part}} = \frac{\frac{25}{3}\pi x}{\frac{175}{3}\pi x} = \frac{1}{7}$$

∴ Ratio of volumes of two parts of cone = 1 : 7

**Question 51.**

A solid metallic sphere of diameter 8 cm is melted and drawn into a cylindrical wire of uniform width. If the length of the wire is 12 m, find its width

**Solution:**

Diameter of sphere radius of sphere=8 cm

Volume of sphere=4 cm

Volume of sphere=4/3π$(4)^3$=4/3π(64) $cm^3$

Length of cylindrical wire, h=12 m=1200 cm

Radius of cylindrical wire=r

Volume of cylindrical wire $=\pi r^2 h=\pi(r)^2 \times 1200\ cm^3$
volume of sphere= volume of cylinder
$4/3\pi(64)=\pi(r)^2 \times 1200$
$4 \times 64/3 \times 1200 = r^2$
$64/(3 \times 3) \times (100)=r^2$
$r=8/3 \times 10=8/30=0.26$ cm
Width of wire = 0.26 cm

**Question 52.**
The total surface area of a solid cylinder is 231 $cm^2$. If the curved surface area of this solid cylinder is 2/3 of its total surface area, find its radius and height
**Solution:**
Let radius of the base of cyclinder = r cm and height = h cm.
Total surface area = $2\pi r (r + h) = 231\ cm^2$.
Curved surface area = $2\pi rh$
Curved surface area =2/3 Total surface area
$2\pi rh=2/3 \times 2\pi r(r+h) \Rightarrow 2\pi rh=2/3 \times 231$
$2\pi rh=154\ cm^2$—-(1)
$2\pi r(r+h)=231$
$2\pi r+2\pi rh=231$
$2\pi r^2+154=231$
$2\pi r^2=231-154=77$
$2 \times 22/7 \times r^2 =77 \Rightarrow r^2=77 \times 7/22 \times 2 \Rightarrow r=7/2$ cm=3.5 cm =radius
From (1)
$2\pi rh=154$
$2 \times 22/7 \times 7/2 \times h =154$
Height=154/22=7 cm

## Long Answer Type Questions [4 Marks]

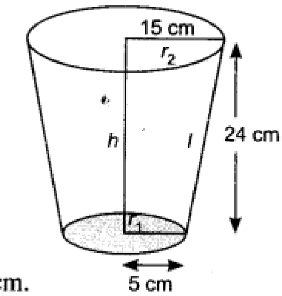
**Question 53.**
Water is flowing through a cylindrical pipe of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.
**Solution:**
Diameter of cylindrical pipe = 2 cm
Radius of cylindrical pipe, r = 2/2 = 1 cm
Rate of flowing of water in 1 sec = 0.4 m/s = 40 cm/sec.
So, volume of water in 1 sec = $\pi r^2 h$
$= \pi \times 1 \times 1 \times 40 = 40\pi\ cm^3$
volume of the water in 1/2 hr=$40\pi \times 1800 = 7200\pi\ cm^3$
Radius of cylindrical tank = 40 cm
Let h' be the rise in level of water in the tank.
So, A.T.Q.
Volume of tank = Volume of flowing water
$\pi \times 40 \times 40 \times h' = 72000\pi$
h'=72000/40 x 40=45 cm
Hence, rise in level of water = 45 cm

**Question 54.**
A bucket open at the top and made up of a metal sheet is in the form of a frustum of a cone. The depth of the bucket is 24 cm and the diameters of its upper and lower circular ends are

30 cm and 10 cm respectively. Find the cost of metal sheet used in it at the rate of rupee 10 per 100 cm2,

**Solution:**

Height of bucket, $h = 24$ cm
Diameter of upper end = 30 cm
So, radius of upper end, $r_2 = 15$ cm
Diameter of lower end = 10 cm
So, radius of lower end, $r_1 = 5$ cm




Now, slant height '$l$' of frustum $= \sqrt{h^2+(r_2-r_1)^2}$

$= \sqrt{24^2+(15-5)^2}$

$= \sqrt{576+100} = \sqrt{676} = 26$ cm.

Curved surface area of frustum (bucket) $= \pi(r_1 + r_2)l + \pi r_1^2$

$= \frac{22}{7}(5+15)\times 26+\frac{22}{7}\times 5\times 5$

$= \frac{22}{7} \times (20 \times 26 + 25) = \frac{22}{7} \times 545 \text{ cm}^2$

Rate of metal sheet used is ₹ 10 per 100 cm$^2$

So, total cost of metal sheet used $= \frac{22}{7}\times 545\times\frac{10}{100} =$ ₹ 171.28

**Question 55.**

Water running in a cylindrical pipe of inner diameter 7 cm, is collected in a container at the rate of 192.5 litres per minute. Find the rate of flow of water in the pipe in km/hr

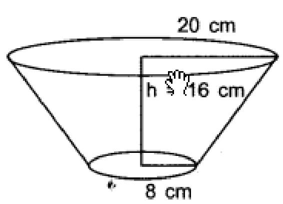
**Solution:**

Diameter of cylindrical pipe = 7 cm
Radius of cylindrical pipe = 7/2 cm
Volume of flowing water in 1 min = 192.5 litre = 192.5 x 1000 cm$^3$ = 192500 cm$^3$
Volume of flowing water in 1 hour = 192500 x 60 cm$^3$
Let 'h' be the length covered by flowing water in cylindrical pipe in 1 hour
Volume of water flowing through cylindrical pipe in 1 hour=$\pi r^2 h$=22/7(7/2)$^2$ x h
192500 x 60 x 7 x 4/22 x 49 =h
h=300000 cm=300000/100000=3 km
Rate of flowing water in 1hr =3km/hr

**Question 56.**

A container open at the top and made up of metal sheet is in the form of a frustum of a cone of height 16 cm with diameters of its lower and upper ends as 16 cm and 40 cm respectively. Find the cost of metal sheet used to make the container, if it costs ? 10 per 100 cm$^2$.

**Solution:**




Radius of upper end of frustum r1 = 20 cm
Radius of lower end of frustum r2 = 8 cm
Height of frustum = 16 cm
Slant height of frustum =l = √h$^2$ + (r1 – r2)$^2$
=√(16)2+ (20-8)$^2$ = √256 +144 = √400 = 20 cm
Curved surface area of frustum open at top
= πt(r1 + r2)l + πr2$^2$
=22/20 (20 + 8)20 + 22/7 (8)$^2$
=22/7[28 x 20 + 64]

$= 22/7[560 + 64] = 22/7 \times 624 \text{ cm}^2$

Total cost of metal sheet used = $10/100 \times 22/7 \times 624 = 196.141$ rupees.

**Question 57.**

A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled ?

**Solution:**

$$\text{Internal diameter of pipe} = 20 \text{ cm}$$

$$\text{Internal radius of pipe} = \frac{20}{2} = 10 \text{ cm}$$

Water flowing at the rate 3 km/h

$\therefore$ In 1 hour, volume of water flowing $= \pi r^2 h = \pi 100(300000) \text{ cm}^3$

$$\text{Radius of cylindrical tank} = \frac{10}{2} = 5 \text{ m} = 500 \text{ cm}$$

Height of cylindrical tank = 2 m = 200 cm

Volume of water collected in cylindrical tank $= \pi r^2 h = \pi(500)^2 \times 200$

$$\text{Time taken to fill the tank} = \frac{\text{Volume of cylindrical tank}}{\text{Volume of water flowing through pipe in 1 hour}}$$

$$= \frac{\pi(500)^2 \times 200}{\pi(100)300000}$$

$$= \frac{250000 \times 200}{30000000} = \frac{50}{30} \text{ hour}$$

$$= 1 \text{ hour } 40 \text{ min.}$$

**Question 58.**

A bucket open at the top is of the form of a frustum of a cone. The diameters of its upper and lower circular ends are 40 cm and 20 cm respectively. If a total of 17600 $\text{cm}^3$ of water can be filled in the bucket, find its total surface area

**Solution:**

$$\text{Radius of upper end} = R_1 = \frac{40}{2} = 20 \text{ cm}$$

$$\text{Radius of lower end} = R_2 = \frac{20}{2} = 10 \text{ cm}$$

Let height = $h$ cm

Volume of bucket = 17600 $\text{cm}^3$

$\Rightarrow \frac{1}{3}\pi h(R_1^2 + R_2^2 + R_1R_2) = 17600$ [$\because$ Bucket is in shape of frustum of cone]

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times h[(20)^2 + (10)^2 + 20 \times 10] = 17600$$

$$\Rightarrow \text{Height of bucket, } h = \frac{17600 \times 7 \times 3}{22 \times 700} = 24 \text{ cm}$$

Now, slant height of bucket, $l = \sqrt{h^2 + (R_1 - R_2)^2}$

$$l = \sqrt{(24)^2 + (10)^2} = \sqrt{676} = 26 \text{ cm}$$

$$\text{Total surface area} = \pi R_2^2 + \pi l(R_1 + R_2)$$

$$= \pi[(10)^2 + 26(20 + 10)]$$

$$= \frac{22}{7} \times 880 \text{ cm}^2 = 2765.714 \text{ cm}^2$$

## 2012

## Short Answer Type Questions I [2 Marks]

**Question 59.**

The volume of a hemisphere is 2425 1/2 $\text{cm}^3$. Find its curved surface area.

**Solution:**

Let the radius of hemisphere be $r$ cm.

$$\text{Volume of hemisphere} = 2425\frac{1}{2} \text{ cm}^3$$
$$\frac{2}{3}\pi r^3 = \frac{4851}{2}$$
$$r^3 = \frac{4851}{2 \times 2\pi} \times 3$$
$$r^3 = \frac{4851 \times 3 \times 7}{2 \times 2 \times 22}$$
$$r = \frac{21}{2} \text{ cm}$$
$$\text{CSA of hemisphere} = 2\pi r^2$$
$$= 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 693 \text{ cm}^2.$$

**Question 60.**

A solid sphere of radius 10.5 cm is melted and recast into smaller solid cones, each of radius 3.5 cm and height 3 cm. Find the number of cones so formed.

**Solution:**

**Solid sphere:** Radius, R = 10.5 cm

**Cone:** Radius, $r$ = 3.5 cm and height, $h$ = 3 cm.

Let solid sphere be melted and recast into '$n$' number of smaller identical cones.

Then, Volume of '$n$' cones = Volume of solid sphere

$$\Rightarrow n \times \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$
$$\Rightarrow \frac{1}{3} \times n \times 3.5 \times 3.5 \times 3 = \frac{4}{3} \times 10.5 \times 10.5 \times 10.5$$
$$\Rightarrow n = \frac{4 \times 10.5 \times 10.5 \times 10.5}{3 \times 3.5 \times 3.5} = 126$$

Hence, number of cones so formed = 126.

**Question 61.**

A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal,then find the ratio of the radius and the height of the conical part

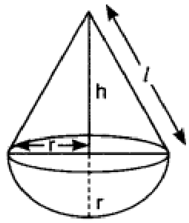
**Solution:**

Let radius of the base = $r$

Heigth of conical part = $h$

Slant height of conical part = $l$

$$\therefore l = \sqrt{h^2 + r^2} \quad \text{...}(i)$$




Curved surface area of hemisphere = Curved surface area of cone

Here, $2\pi r^2 = \pi r l$

$$\Rightarrow l = 2r \quad \text{...}(ii)$$

From $(i)$ & $(ii)$, we have

$$2r = \sqrt{h^2 + r^2}$$
$$\Rightarrow 4r^2 = h^2 + r^2 \Rightarrow h^2 = 3r^2$$
$$\Rightarrow \frac{r^2}{h^2} = \frac{1}{3} \Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}}$$

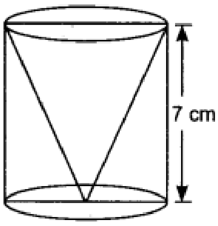
$\therefore$ Required ratio of radius and height of conical part = $1 : \sqrt{3}$

## Short Answer Type Questions II [3 Marks]

**Question 62.**

From a solid cylinder of height 7 cm and base diameter 12 cm, a conical cavity of same height and same base diameter is hollowed out. Find the total surface area of the remaining solid.

**Solution:**




Radius of cylinder = Radius of cone, $r = \frac{12}{2} = 6$ cm

Height of cylinder = Height of cone, $h = 7$ cm

$\therefore$ Slant height of cone, $l = \sqrt{r^2 + h^2}$

$= \sqrt{6^2 + 7^2} = \sqrt{85}$ cm

Total Surface Area of remaining solid

= Curved Surface Area of cone + Curved Surface Area of cylinder + Area of base

$= \pi rl + 2\pi rh + \pi r^2 = \pi r(l + 2h + r)$

$= \frac{22}{7} \times 6 \times (\sqrt{85} + 2 \times 7 + 6) = \frac{132}{7} \times (9.21 + 14 + 6) = 550.99 \text{ cm}^2$

**Question 63.**

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, then find the radius and slant height of the heap.

**Solution:**

Let,Height of cylinder, H=32 cm

Radius of cylinder, R=18 cm

Height of cone, h =24 cm

Radius of cone, r =?

Volume of bucket=Volume of heap

$\pi R^2 H = 1/3 \pi r^2 h$

$18 \times 18 \times 32 = 1/3 \times r^2 \times 24$

$18 \times 18 \times 32/8 = r^2$

$1296 = r^2$

Radius of heap, $r = \sqrt{1296} = 36$ cm

slant height of heap, $l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 36^2} = \sqrt{576 + 1296} = \sqrt{1872} = 43.27$ cm

**Question 64.**

A hemispherical bowl of internal radius 9 cm is full of water. Its contents are emptied in a cylindrical vessel of internal radius 6 cm. Find the height of water in the cylindrical vessel.

**Solution:**

Hemispherical Bowl:

Radius, R = 9 cm

Cylindrical Vessel:

Radius, r = 6 cm

Let, height = h cm

Content of hemispherical bowl is completely transferred to cylindrical vessel.

So, Volume of cylindrical vessel = Volume of hemispherical bowl

=> $\pi r^2 h = 2/3 \pi R^3$

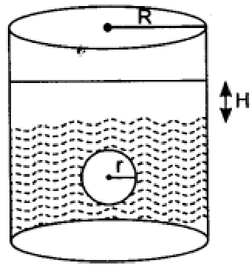
$6 \times 6 \times h = 2/3 \times 9 \times 9 \times 9$ => $h = 2 \times 9 \times 9 \times 9/3 \times 6 \times 6 = 13.5$ cm

.’. Height of water in cylindrical vessel = 13.5 cm

**Question 65.**

A sphere of diameter 6 cm is dropped into a cylindrical vessel, partly filled with water, whose diameter is 12 cm. If the sphere is completely submerged in water, by how much will the surface of water be raised in the cylindrical vessel?

**Solution:**




Diameter of sphere is 6 cm, so radius of sphere i.e. r =6/2 = 3 cm Diameter of the cylinder is 12 cm.

.'. Radius (R) =12/2 = 6 cm

Let the height of cylinder is H after increasing the water level.

Now, A.T.Q., Volume of sphere = Volume of cylinder

$4/3\pi r^3=\pi R^2H$

$4/3 \times (3)^3=(6)^2.H$

H=4x3x3x3/3x6x6=1cm

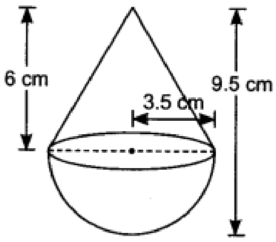
Water level will be raised by 1 cm.

## Long Answer Type Questions [4 Marks]

**Question 66.**

A solid is in the shape of a cone surmounted on a hemisphere, the radius of each of them being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid

**Solution:**




Let r be the common radius of each of them and h be the height of the cone.

.'. r = 3.5 cm, h = 6 cm

Volume of the solid = Volume of the cone +Volume of the hemisphere

$=1/3\pi r^2h+2/3\pi r^3$

$=1/3\times22/7\times3.5\times3.5\times6+2/3\times22/7\times3.5\times3.5\times3.5$

$=1/3\times22/7 \times 3.5 \times 3.5 [6 + 2 \times 3.5]$

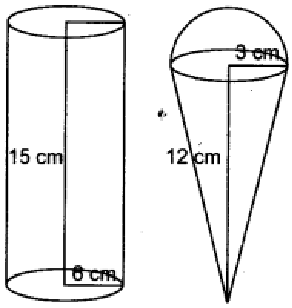
$= 1/3 \times 22 \times 0.5 \times 3.5 [6 + 7]$

$=1/3 \times 22 \times 0.5 \times 3.5 \times 13 = 166.83\ cm^3$

**Question 67.**

A container shaped like a right circular cylinder having base radius 6 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled into cones of height 12 cm and radius 3 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

**Solution:**




Let radius of cylinder, R = 6 cm

Height, H = 15 cm

Radius of cone, r = 3 cm

Height of cone, h = 12 cm

Number of cones=Volume of cylinder/Volume of cone

$\pi R^2H/1/3\pi r^2h=3 \times 6 \times 6 \ \times 15/3 \times 3 \times 12$

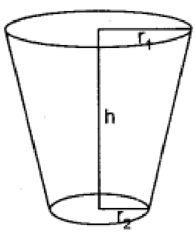
n = 15

Number of cones filled with ice-cream = 15

**Question 68.**

A bucket is in the form of a frustum of a cone and it can hold 28.49 litres of water. If the radii of its circular ends are 28 cm and 21 cm, find the height of the bucket.

**Solution:**




r1=28 cm,r2=21 cm,h=?

volume=28.49l

$=28.49 \times 1000 =28490\ cm^3$

$v=1/3\pi(r1^2+r2^2+r1r2\ )h$

$28490=22/3 \times 7[(28)^2+(21)^2+28 \times 21]h$

$28490 \times 3 \times 7/22=(784+441+588)h$

27195=1813h

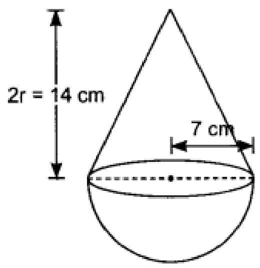
h=27195/1813=15 cm

Height of bucket=15 cm

**Question 69.**

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid

**Solution:**




Let r be the common radius of each of them and h be the height of the cone.

r = 7 cm, h = 2r = 2×7 = 14cm

Volume of the solid = Volume of the cone +Volume of the hemisphere

=1/3πr²h+2/3πr³=1/3xπxr²[h+2r]

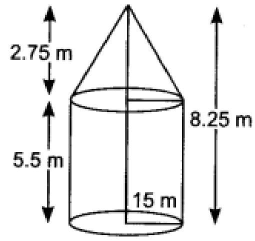
=1/3×22/7×7[14+14]

=1/3x22x7x28=1437.33 cm³

**Question 70.**

A military tent of height 8.25 m is in the form of a right circular cylinder of base diameter 30 m and height 5.5 m surmounted by a right circular cone of same base radius. Find the length of the canvas use in making the tent, if the breadth of the canvas is 1.5 m.

**Solution:**




Base radius, $r = \frac{30}{2}$ m = 15 m

H = height of cylindrical part = 5.5 m

∴ Height of conical part, $h$ = 8.25 m – 5.5 m = 2.75 m

Slant height of conical part, $l = \sqrt{h^2 + r^2}$

$l = \sqrt{(2.75)^2 + 15^2} = 15.25$ m

Total surface area of tent = curved area of conical part + curved area of cylindrical part

$= \pi rl + 2\pi rh$

$= \pi \times 15 \times 15.25 + 2\pi \times 15 \times 5.5$

$= 228.75\pi + 165\pi = 393.75\pi$

$= 393.75 \times \frac{22}{7} = 1237.5 \text{ m}^2$

∴ Area of canvas = 1237.5 $m^2$

Breadth of canvas = 1.5 m

Let 'L' be the length of canvas.

A.T.Q. Length × Breadth = Total Surface Area of tent

L × 1.5 = 1237.5

Length of canvas, $L = \frac{1237.5}{1.5} = 825$ m.

**Question 71.**

A hemispherical tank, full of water, is emptied by a pipe at the rate of litres per sec. How much time will it take to empty half the tank if the diameter of the base ofthe tank is 3 m?

**Solution:**

Diameter of hemisphere = 3 m

Radius of hemisphere = $\frac{3}{2}$ m

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{99}{14} \text{ m}^3$

$= \frac{99}{14} \times 1000 = \frac{99000}{14}\ l$

Half the volume of tank $= \frac{1}{2} \times \frac{99000}{14} = \frac{24750}{7}\ l$

Time taken to empty $\frac{25}{7} l$ = 1 s

Time taken to empty 1 $l = \frac{7}{25}$ s

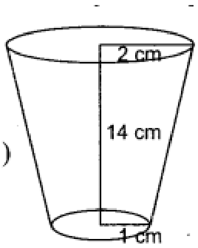
Time taken to empty $\frac{24750}{7}\ l = \frac{24750}{7} \times \frac{7}{25} = 990$ s.

**Question 72.**

A drinking glass is in the shape of the frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass

**Solution:**

Let $r_1$ = radius of upper end = 2 cm
$r_2$ = radius of lower end = 1 cm
$h$ = height of frustum = 14 cm




$$\text{Capacity of glass} = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2) = \frac{1}{3} \times \frac{22}{7} \times 14(2^2 + 1^2 + 2 \times 1)$$

$$= \frac{44}{3}(4 + 1 + 2) = \frac{44}{3} \times 7 = \frac{308}{3} \text{ cm}^3.$$

**Question 73.**

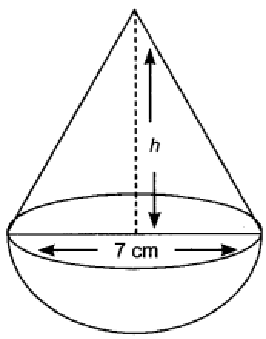
A toy is in the shape of a cone mounted on a hemisphere of same base radius. If the volume of the toy is 231 cm3 and its diameter is 7 cm, then find the height of the toy

**Solution:**

Given, $d = 7$ cm, $r = \frac{7}{2}$ cm

Let the height of cone is $h$ cm, Volume of the toy = 231 $\text{cm}^3$

Volume of the toy = Volume of cone + Volume of hemisphere




$$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = 231 \text{ cm}^3$$

$$\frac{1}{3}\pi r^2(h + 2r) = 231$$

$$\frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left(h + 2 \times \frac{7}{2}\right) = 231$$

$$\frac{154}{12}(h + 7) = 231$$

$$h + 7 = \frac{231 \times 12}{154}$$

$$h + 7 = 18$$

$$h = 18 - 7$$

$$h = 11 \text{ cm} = \text{Height of cone}$$

$$\text{Height of the toy} = \text{Height of the cone} + \text{Radius of hemisphere}$$

$$= 11 + \frac{7}{2} = \frac{29}{2} = 14.5 \text{ cm}$$

**Question 74.**

The radii of internal and external surface of a hollow spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid cylinder of diameter 14 cm. Find the height of the cylinder

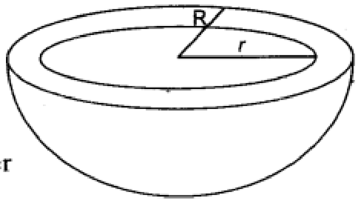
**Solution:**

It is given that r = 3 cm and R = 5 cm, where R–>radius of external surface, r–>radius of internal surface, Diameter of cylinder = 14 cm

Radius of cylinder = $\frac{14}{2} = 7$ cm

Let the height of cylinder is $h$ cm.

Now, volume of hollow spherical shell = volume of cylinder




$$\therefore \quad \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3 = \pi r^2 h$$

$$\frac{2}{3}\pi(R^3 - r^3) = \pi r^2 h$$

$$\frac{2}{3} \times \frac{22}{7}(5^3 - 3^3) = \frac{22}{7} \times 7 \times 7 \times h$$

$$\frac{2}{3} \times \frac{22}{7} \times 98 = 154 \times h$$

$$h = \frac{44 \times 98}{3 \times 7 \times 154}$$

$$h = \frac{4}{3}\text{cm}$$

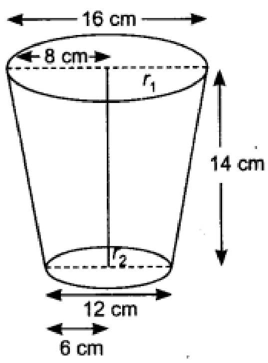
Height of cylindrical, $h$ = 1.3 cm (approx.)

**Question 75.**

A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 16 cm and 12 cm. Find the capacity of the glass

**Solution:**

$r_1 = \frac{16}{2} = 8$ cm, $r_2 = \frac{12}{2} = 6$ cm




$$\text{Capacity of glass} = \text{Volume of frustum of cone}$$

$$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3}\pi h[(8)^2 + (6)^2 + 8 \times 6]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14[64 + 36 + 48]$$

$$= \frac{44}{3} \times 148 = \frac{6512}{3} = 2170\frac{2}{3}\text{cm}^3$$

**2011**

**Short Answer Type Questions I [2 Marks]**

**Question 76.**

Two cubes, each of side 4 cm are joined end to end. Find the surface area of the resulting cuboid

**Solution:**

Length of resulting cuboid, l = 4 cm + 4 cm = 8 cm

Breadth, b = 4 cm, Height, h = 4 cm

Surface area of cuboid = 2(lb + bh + hl)

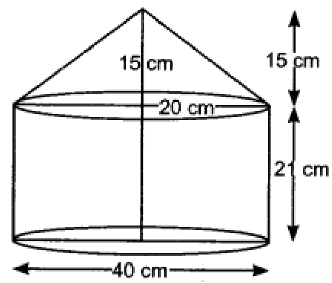
= 2 (8 x 4 + 4 x 4 + 8 x 4) = 160 $cm^2$

**Question 77.**

A toy is in the shape of a solid cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 21 cm and 40 cm respectively, and the height of cone is 15 cm, then find the total surface area of the toy

**Solution:**

Let H be the height of cylinder.
and $h$ be the height of cone
$r$ be the radius of cylinder and cone.
H = 21 cm; $h$ = 15 cm, $r$ = 20 cm



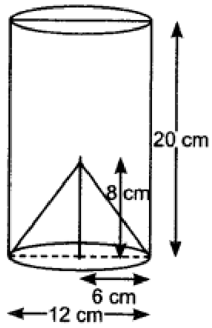

$$\text{Slant height of cone, } l = \sqrt{r^2+h^2} = \sqrt{20^2+15^2} = \sqrt{400+225} = \sqrt{625} = 25 \text{ cm}$$

$$\begin{aligned}\text{Total surface area of toy} &= \text{Curved surface area of cylinder} + \text{Curved surface area of cone} \\ &= 2\pi rH + \pi rl = \pi r(2H + l) \\ &= 3.14 \times 20 \times (2 \times 21 + 25) \\ &= 3.14 \times 20 \times 67 = 4207.6 \text{ cm}^2\end{aligned}$$

**Question 78.**

From a solid cylinder of height 20 cm and diameter 12 cm, a conical cavity of height 8 cm and radius 6 cm is hollowed out. Find the total surface area of the remaining

**Solution:**




Let height of cylinder be H = 20 cm

$$\text{Radius of cylinder be } r = \frac{12}{2} \text{ cm} = 6 \text{ cm}$$

$$\text{height of cone, } h = 8 \text{ cm}$$

$$\text{radius of cone, } r = 6 \text{ cm}$$

$$\text{slant height of cone, } l = \sqrt{r^2+h^2} = \sqrt{6^2+8^2} = \sqrt{100}$$

$$l = 10 \text{ cm}$$

$$\begin{aligned}\text{Total surface area of remaining solid} &= \text{Curved surface area of cylinder} + \text{Curved surface area of cone} + \text{Area of circle} \\ &= 2\pi rH + \pi rl + \pi r^2 \\ &= \pi r(2H + l + r) \\ &= \frac{22}{7} \times 6 \times (2 \times 20 + 10 + 6) \\ &= \frac{22}{7} \times 6 \times 56 = 1056 \text{ cm}^2\end{aligned}$$

**Question 79.**

A cone of height 20 cm and radius of base 5 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the diameter of the sphere

**Solution:**

Height of the cone, H = 20 cm

Radius of base, R = 5 cm Let radius of the sphere be 'r'.

A.T.Q. Volume of the cone = Volume of the sphere

$1/3\pi R^2H=4/3\pi r^3$

5 x 5 x 20 =$4xr^3$

=>5x5x20/4 = $r^3$

=> $r^3$ = 5x5x5

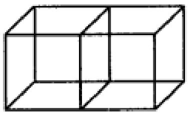
=> r = 5 cm

Diameter of the sphere = 2 x 5 = 10 cm

**Question 80.**

Two cubes each of volume 27 cm3 are joined end to end to form a solid. Find the surface area of the resulting cuboid

**Solution:**



Let 'a' be the side of each cube of volume 27 $cm^3$ then $a^3 = 27$

=> a = 3 cm

When two cubes are joined end to end to form a solid then cuboid will be formed.

length of cuboid, l = 6 cm = 3 cm + 3 cm

breadth of cuboid, b = 3 cm

height of cuboid, h = 3 cm

Total surface area of the cuboid = 2 (lb + bh + hL) = 2(6 x3 + 3×3 + 3×6)

= 2(18 + 9 + 18) = 2 x 45 = 90 $cm^2$

**Question 81.**

The dimensions of a metallic cuboid are 100 cm x 80 cm x 64 cm. It is melted and recast into a cube. Find the surface area of the cube

**Solution:**

Dimensions of the metallic cuboid are 100 cm x 80 cm x 64 cm

Metallic cuboid is recasted into a cube.

Then, Volume of cuboid = Volume of cube

=> 100 x 80 x 64 = $a^3$ (where a is the side of cube)

=> $\sqrt[3]{100 \times 80 \times 64} = a$

$\sqrt[3]{512000} = a$

> a = 80 cm

Now, Surface area of the cube = $6a^2 = 6(80)^2$ = 6 x 80 x 80 = 38400 $cm^2$

**Question 82.**

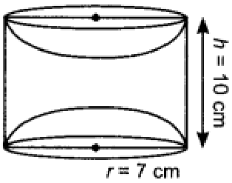
A wooden article was made by scooping out a hemisphere of radius 7 cm, from each end of a solid cylinder of height 10 cm and diameter 14 cm. Find the total surface area of the article

**Solution:**

Given, radius of cylinder, $r = 7$ cm

height of cylinder, $h = 10$ cm

radius of hemisphere, $r = 7$ cm




Total surface area of the article = Curved surface area of cylinder + Curved surface area of 2 hemispheres

$$= 2\pi rh + 2 \times 2\pi r^2 = 2\pi r(h + 2r)$$

$$= 2 \times \frac{22}{7} \times 7 \times (10 + 2 \times 7)$$

$$= 2 \times 22 \times 24 = 1056 \text{ cm}^2$$

## Short Answer Type Questions II [3 Marks]

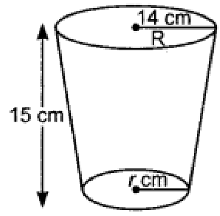
**Question 83.**

The radii of the circular ends of a bucket of height 15 cm are 14 cm and r cm (r < 14 cm). If the volume of bucket is 5390 cm3, then find the value of r

**Solution:**

Given, Radii of bucket are R $= 14$ cm and $r$ cm.

Height of bucket $h = 15$ cm




$$\therefore \quad \text{Volume of the bucket} = \frac{1}{3}\pi h[R^2 + R \times r + r^2]$$

$$\Rightarrow \quad 5390 = \frac{1}{3} \times \frac{22}{7} \times 15[196 + 14r + r^2]$$

$$\Rightarrow \quad \frac{5390 \times 7}{22 \times 5} = 196 + 14r + r^2$$

$$\Rightarrow \quad 343 - 196 = 14r + r^2$$

$$\Rightarrow \quad r^2 + 14r - 147 = 0$$

$$\Rightarrow \quad r^2 + 21r - 7r - 147 = 0$$

$$\Rightarrow \quad r(r + 21) - 7(r + 21) = 0$$

$$\Rightarrow \quad (r - 7)(r + 21) = 0$$

$$\Rightarrow \quad r = 7 \text{ or } r = -21$$

$r = -21$ is rejected as radius can never be negative

$$\therefore \quad r = 7 \text{ cm}$$

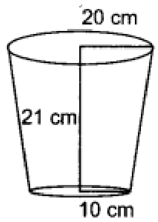
**Question 84.**

An open metal bucket is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the cost ofmilk which can completely fill the bucket at ? 30 per litre

**Solution:**

Here Radii of bucket are R $= 20$ cm, $r = 10$ cm

Height of bucket, $h = 21$ cm




$$\text{Capacity of bucket} = \frac{1}{3}\pi h[R^2 + r^2 + Rr]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 21 \times [(20)^2 + (10)^2 + 20 \times 10]$$

$$= 22\,[400 + 100 + 200]$$

$$= 22 \times 700 = 15400 \text{ cm}^3$$

$$= \frac{15400}{1000}\, l = 15.4\, l \qquad \left[\because 1 \text{ cm}^3 = \frac{1}{1000} l\right]$$

Total cost of milk at the rate of ₹ 30 per litre = ₹ 30 × 15.4 = ₹ 462.00

**Question 85.**

From a solid cylinder of height 14 cm and base diameter 7 cm, two equal conical holes each of radius 2.1 cm and height 4 cm are cut off. Find the volume of the remaining solid

**Solution:**

Cylinder:

Height, H = 14 cm; Diameter = 7 cm; Radius, R = 7/2 cm

Cone:

height, h = 4 cm; radius, r = 2.1 cm

Volume of remaining solid = Volume of cylinder – Volume of 2 cones

$= \pi R^2 H - 2 \times 1/3 \pi r^2$

$=22/7 \times 7/2 \times 7/2 \times 14 - 2 \times 1/3 \times 22/7 \times 2.1 \times 2.1 \times 4$

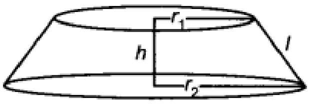
$= 539 - 36.96 = 502.04$ cm$^3$

**Question 86.**

The radii of the circular ends of a solid frustum of a cone are 18 cm and 12 cm respectively and its height is 8 cm. Find its total surface area

**Solution:**

Given that: $r_1 = 12$ cm; $r_2 = 18$ cm, $h = 8$ cm




$$\text{Slant height, } l = \sqrt{h^2 + (r_2 - r_1)^2}$$
$$= \sqrt{8^2 + (18-12)^2}$$
$$= \sqrt{64+36} = 10 \text{ cm}$$

Now, Total surface area of frustum $= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

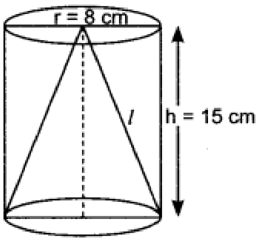
$$= \pi[(r_1 + r_2)l + r_1^2 + r_2^2]$$
$$= \frac{22}{7}[(12 + 18) \times 10 + 12^2 + 18^2]$$
$$= \frac{22}{7}[300 + 144 + 324]$$
$$= \frac{22}{7} \times 768 = \frac{16896}{7} = 2413.71 \text{ cm}^2$$

## Long Answer Type Questions [4 Marks]

### Question 87.

From a solid cylinder whose height is 15 cm and diameter 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid

**Solution:**




Here, height of cylinder = height of cone, $h = 15$ cm

Base radius of cylinder = Base radius of cone, $r = 8$ cm

Let '$l$' be slant height of cone. Using,

$$l^2 = r^2 + h^2$$
$$= 8^2 + 15^2 = 289$$
$$\Rightarrow \quad l = \sqrt{289} = 17 \text{ cm}$$

Now, Total Surface Area of solid = Curved surface area of cylinder + Curved surface area of cone + Area of circular base

$$= 2\pi rh + \pi rl + \pi r^2$$
$$= \pi r(2h + l + r)$$
$$= 3.14 \times 8[2 \times 15 + 17 + 8]$$
$$= 25.12 \times 55 = 1381.6 \text{ cm}^2.$$

### Question 88.

Water is flowing at the rate of 10 km/hour through a pipe of diameter 16 cm into a cuboidal tank of dimensions 22 m x 20 m x 16 m. How long will it take to fill the empty tank

**Solution:**

**Cuboid:** Length; $l = 22$ m

Breadth, $b = 20$ m

Height, $h = 16$ m

**Cylindrical pipe:** Diameter = 16 cm

So, Radius, $r = 8$ cm $= 0.08$ m

Volume of cuboidal tank $= lbh$

$$= 22 \times 20 \times 16 \text{ m}^3$$

Length of water covered by pipe in 1 hr, $h' = 10$ km $= 10000$ m

Volume of water filled by pipe in 1 hr $= \pi r^2 h'$

$$= \frac{22}{7} \times 0.08 \times 0.08 \times 10000 \text{ m}^3$$

Time taken to fill 1 $m^3$ volume of water by pipe $= \frac{7}{22 \times 0.08 \times 0.08 \times 10000}$ hrs

Total time taken by pipe to fill cuboidal tank completely $= \frac{7 \times 22 \times 20 \times 16}{22 \times 0.08 \times 0.08 \times 10000}$ hrs

$$= 35 \text{ hrs.}$$

## Question 89.

Water is flowing at the rate of 15 km/hour through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm?

**Solution:**

Let the level of water rise in the tank in $x$ hours

Length of the water flow in $x$ hours = $15000x$ metres

Diameter of the pipe = 14 cm

$\therefore$ Radius, $r = \frac{14}{2} = 7 \text{ cm} = \frac{7}{100} \text{ m} = 0.07 \text{ m}$

Volume of water = $\pi r^2 h$

Volume of water flow in $x$ hours in the pond = $\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000x \text{ m}^3$

Volume of cuboidal pond with water level of height 21 cm, i.e. $\frac{21}{100}$ m = $50 \times 44 \times \frac{21}{100} \text{ m}^3$

$= l \times b \times h$

Volume of cuboidal pond = Volume of water flow in $x$ hrs

According to questions,

$$50 \times 44 \times \frac{21}{100} = \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000x$$

$$\Rightarrow \quad 22 \times 21 = \frac{154 \times 15}{10} x$$

$$\Rightarrow \quad x = \frac{22 \times 21 \times 10}{154 \times 15} = 2 \text{ hrs}$$

Hence, the level of water in the pond will rise by 21cm in 2 hours.

## Question 90.

A farmer connects a pipe of internal diameter 20 cm, from a canal into a cylindrical tank in his field, which is 10 m in diameter and 4 m deep. If water flows through the pipe at the rate of 5 km/hour, in how much time will the tank be filled?

**Solution:**

**Cylindrical tank:** Radius; R = 5 m

Height, H = 4 m

**Cylindrical pipe:** Radius, $r$ = 10 cm = 0.1 m

Now, volume of cylindrical tank = $\pi R^2 H = \frac{22}{7} \times (5)^2 \times 4 = 314.28 \text{ m}^3$

Now, length covered by cylindrical pipe in 1 hr, $h$ = 5 km = 5000 m

Volume of water flows through pipe in 1 hr = $\pi r^2 h$

$= \frac{22}{7} \times 0.1 \times 0.1 \times 5000 \text{ m}^3 = 157.14 \text{ m}^3$

Time taken to fill water of volume 1 m$^3$ = $\frac{7}{22 \times 0.1 \times 0.1 \times 5000}$ hr = 0.0064 hrs.

Total time taken to fill cylindrical tank completely = $\frac{314.28}{157.14}$ = 2 hrs = 120 mins

## Question 91.

Water is flowing at the rate of 6 km/h through a pipe of diameter 14 cm into a rectangular tank which is 60 m long and 22 m wide. Determine the time in which the level of the water in the tank will rise by 7 cm.

**Solution:**

Radius of pipe = 7 cm = $\frac{7}{100}$ m.

Volume of water flowing through the cylindrical pipe in one hour at rate of 6 km/hr

$= \pi \left(\frac{7}{100}\right)^2 \times 6000 \text{ m}^3$

$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 6000 = \frac{462}{5} \text{ m}^3 = 92.4 \text{ m}^3$

Level of water to be raised in 60 m × 22 m water tank = 7 cm = $\frac{7}{100}$ m.

The volume of water added in the water tank $= 60 \times 22 \times \frac{7}{100} = \frac{462}{5}$ m$^3$ = 92.4 m$^3$

Let the required time to raise the water level in the tank by 7 cm be '$t$' hours.

$$\therefore \quad \left(\frac{462}{5}\right) \times t = \frac{462}{5}$$

$$t = 1$$

Hence, the level of water in the tank will rise by 7 cm in 1 hr.

**Question 92.**

A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted to form a cone of base diameter 8 cm. Find the height and the slant height of the cone.

**Solution:**

| Hollow sphere | Cone |
|---|---|
| Internal radius, $r = 2$ cm | Base radius, $r_1 = 4$cm |
| External radius, $R = 4$ cm | Let height be $h$ and slant height be '$l$' |

When hollow sphere is melted to form a cone then

Volume of cone = Volume of sphere

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h = \frac{4}{3}\pi(R^3 - r^3)$$

$$\Rightarrow \quad \frac{1}{3}\pi \times 4^2 \times h = \frac{4}{3}\pi(4^3 - 2^3)$$

$$\Rightarrow \quad 4 \times 4 \times h = 4 \times (64 - 8)$$

$$\Rightarrow \quad 4 \times h = 56$$

$$\Rightarrow \quad h = 14 \text{ cm} = \text{Height of cone}$$

Now, slant height of cone, $l = \sqrt{h^2 + r_1^2}$

$$= \sqrt{14^2 + 4^2} = \sqrt{196 + 16} = \sqrt{212} = 2\sqrt{53} \text{ cm}$$

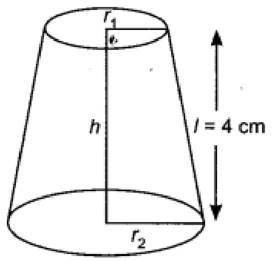
## 2010

### Very Short Answer Type Questions [1 Mark]

**Question 93.**

The slant height of a frustum of a cone is 4 cm and the perimeters (circumferences) of its circular ends are 18 cm and 6 cm respectively. Find the curved surface area of the frustum

**Solution:**




Slant height '$l$' of the frustum of a cone is 4 cm.

Perimeter of circular ends are 18 cm and 6 cm.

Let $r_1$ & $r_2$ be the radius of circular ends,

then, A.T.Q.

$$2\pi r_1 = 6$$

$$r_1 = \frac{6 \times 7}{2 \times 22} = \frac{21}{22} \text{ cm}$$

and

$$2\pi r_2 = 18$$

$$r_2 = \frac{18 \times 7}{2 \times 22} = \frac{63}{22} \text{ cm}$$

Curved Surface Area of frustum of cone $= \pi l (r_1 + r_2)$

$$= \frac{22}{7} \times 4 \times \left[\frac{21}{22} + \frac{63}{22}\right] = \frac{22}{7} \times 4 \times \frac{84}{22} = 48 \text{ cm}^2$$

**Question 94.**

The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, write the height of the frustum

**Solution:**

Slant height of the frustum of a cone is $l = 5$ cm

Let $r_1$ & $r_2$ be the radius of circular ends, then

$$r_1 - r_2 = 4 \text{ cm}$$

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

Now, $l^2 = h^2 + (r_1 - r_2)^2$, where 'h' is the height of the frustum.

$\Rightarrow$ $5^2 = h^2 + 4^2$

or $25 = h^2 + 16$

$\Rightarrow$ $25 - 16 = h^2$

$9 = h^2 \Rightarrow$ Height of frustum, $h = 3$ cm

**Question 95.**

The slant height of a frustum of a cone is 10 cm. If the height of the frustum is 8 cm, then find the difference of the radii of its two circular ends

**Solution:**

$\because$ Given, $l = 10$ cm. So, $l = \sqrt{h^2 + (r_2 - r_1)^2}$ where $l$ = slant height, $h$ = height

$r_1$ and $r_2$ are the radius of circular ends.

$\Rightarrow$ $\sqrt{64 + (r_2 - r_1)^2} = 10$

$\Rightarrow$ $(r_2 - r_1)^2 = 100 - 64$

$\Rightarrow$ $(r_2 - r_1)^2 = 36$

$\Rightarrow$ $r_2 - r_1 = 6$ cm

$\therefore$ Difference of radii of its two circular ends = 6 cm

## Short Answer Type Question II [3 Marks]

**Question 96.**

The rain-water collected on the roof of a building, of dimensions 22 m X 20 m, is drained into a cylindrical vessel having base diameter 2 m and height 3.5 m. If the vessel is full up to the brim, find the height of rain-water on the roof

**Solution:**

Let the height of rain water on the roof = $h$ cm. Then,

A.T.Q. Volume of rain falling on the roof = Volume of cylindrical vessel

$\Rightarrow$ $22 \times 20 \times h = \pi \times (1)^2 \times 3.5$

$$h = \frac{22 \times 35}{7 \times 10 \times 22 \times 20} \text{ m} = 0.025 \text{ m} = 2.5 \text{ cm}$$

$\therefore$ Height of rain-water on roof = 2.5 cm

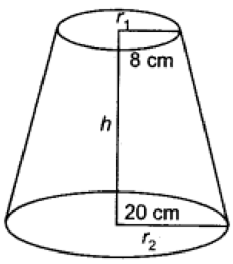
## Long Answer Type Questions [4 Marks]

**Question 97.**

A milk container is made of metal sheet in the shape of frustum of a cone whose volume is 10459 3/7 cm³ The radii of its lower and upper circular ends are 8 cm and 20 cm respectively. Find the cost of metal sheet used in making the container at the rate of rupee 1.40 per square centimeter.

**Solution:**

Radius of lower circular end of frustum, $r_1 = 8$cm
Radius of upper circular end of frustum, $r_2 = 20$ cm.
Let height of frustum be $h$.




$$\text{Volume of frustum} = \frac{1}{3}\pi(r_1^2 + r_1r_2 + r_2^2)h$$

$$\Rightarrow \quad 10459\frac{3}{7} = \frac{1}{3} \times \frac{22}{7}(8^2 + 8 \times 20 + 20^2)h$$

$$\Rightarrow \quad \frac{73216}{7} = \frac{1}{3} \times \frac{22}{7} \times (64 + 160 + 400) \times h$$

$$\frac{73216}{7} = \frac{1}{3} \times \frac{22}{7} \times 624 \times h$$

$$\Rightarrow \quad \therefore \text{ Height of frustum, } h = \frac{73216}{7} \times \frac{3 \times 7}{22 \times 624} = 16 \text{ cm}$$

$$\text{Slant height, } l = \sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{16^2 + (20-8)^2}$$

$$= \sqrt{256 + 144} = \sqrt{400} = 20 \text{ cm}$$

Area of metal sheet used in making the container = Curved surface area of cone + Area of circular base

$$= \pi(r_1 + r_2)l + \pi r_1^2$$

$$= \frac{22}{7}(8 + 20) \times 20 + \frac{22}{7} \times 8^2$$

$$= \frac{22}{7} \times 28 \times 20 + \frac{22}{7} \times 64$$

$$= \frac{22}{7} \times (560 + 64)$$

$$= \frac{22}{7} \times 624 = \frac{13728}{7} \text{ cm}^2$$

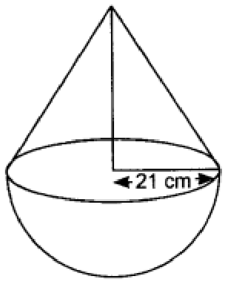
So, Cost of making the container $= ₹\left(\frac{13728}{7} \times 1.4\right) = ₹\ 2745.60$

**Question 98.**

A toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of base of the cone is 21 cm and its volume is 2/3 of the volume of the hemisphere, calculate the height of the cone and the surface area of the toy.

**Solution:**

Radius of the base of cone, $r = 21$ cm
Let '$h$' is the height of cone. Then,




$$\text{Volume of cone} = \frac{2}{3} \text{ Volume of hemisphere}$$

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{2}{3}\pi r^3$$

$$\Rightarrow \quad h = \frac{4}{3}r$$

$$\Rightarrow \quad \text{Height of cone, } h = \frac{4}{3} \times 21 = 28 \text{ cm}$$

$$\therefore \quad \text{Slant height of cone, } l^2 = h^2 + r^2, \text{ where } l \text{ is the slant height of cone}$$

$$= (28)^2 + (21)^2$$

$$= 784 + 441 = 1225$$

$$\Rightarrow \quad \therefore \quad l = \sqrt{1225} = 35 \text{ cm}$$

Surface area of the toy = Curved surface area of cone + Curved surface area of hemisphere

$$= \pi rl + 2\pi r^2$$

$$= \frac{22}{7} \times 21 \times 35 + 2 \times \frac{22}{7} \times 21 \times 21$$

$$= 2310 + 2772 = 5082 \text{ cm}^2$$

**Question 99.**

The difference between the outer and inner curved surface areas of a hollow right circular cylinder, 14 cm long, is 88 cm². If the volume of metal used in making the cylinder is 176 cm³, find the outer and inner diameters of the cylinder

**Solution:**

Let outer radius of the cylinder = R

Inner radius = $r$

According to question,

$$2\pi Rh - 2\pi rh = 88$$
$$2\pi h(R - r) = 88$$
$$R - r = \frac{88}{2\pi h} = \frac{88 \times 7}{2 \times 22 \times 14}$$
$$\Rightarrow \quad R - r = 1 \text{ cm} \quad ...(i)$$

Also, $\quad$ Volume of cylinder $= \pi(R^2 - r^2)h$

$$176 = \frac{22}{7}[R^2 - r^2] \times 14$$
$$176 = 44[R^2 - r^2]$$
$$\frac{176}{44} = R^2 - r^2$$
$$4 = (R - r)(R + r)$$
$$4 = 1(R + r)$$
$$R + r = 4 \quad ...(ii)$$

Add $(i)$ and $(ii)$, we get $\quad R = 2.5$ cm and $r = 1.5$ cm

$\therefore \quad 2R = 5$ cm and $r = 3$ cm

$\therefore \quad$ Outer diameter = 2R = 5 cm, Inner diameter = $2r$ = 3 cm

**Question 100.**

The surface area of a solid metallic sphere is 616 cm². It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed.

**Solution:**

Let the radius of sphere = $r$ cm

Surface area of sphere = 616 cm$^2$. So,

$$4\pi r^2 = 616$$
$$r^2 = \frac{616}{4\pi} = \frac{616 \times 7}{4 \times 22} = 49$$
$$r = 7 \text{ cm}$$

Let the radius of recasted cone = R cm

Height of cone = 28 cm

A.T.Q. $\quad$ Volume of sphere = Volume of cone

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 h$$
$$\frac{4}{3} \times 7 \times 7 \times 7 = \frac{1}{3} \times R^2 \times 28$$
$$\frac{4 \times 7 \times 7 \times 7}{28} = R^2$$
$$R^2 = 7 \times 7$$
$$R = 7 \text{ cm}$$

Diameter of base of cone = 2R = 2 × 7 = 14 cm

**Question 101.**

A container, open at the top, and made of a metal sheet, is in the form of a frustum of a cone of height 24 cm with radii of its lower and upper ends as 7 cm and 14 cm respectively. Find the cost of milk which can completely fill the container at the rate of rupee 25 per litre. Also, find the area of the metal sheet used to make the container.

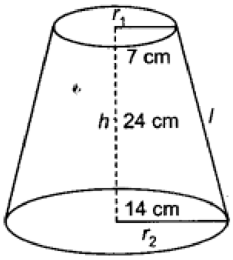
**Solution:**

Given, $r_1 = 7$ cm, $r_2 = 14$ cm, $h = 24$ cm

$\therefore$ Slant height $(l) = \sqrt{(h)^2 + (r_2 - r_1)^2}$

$= \sqrt{(24)^2 + (14-7)^2}$ cm

$= \sqrt{576 + 49}$ cm $= \sqrt{625}$ cm $= 25$ cm




Volume of the milk which will completely fill the container

$$= \frac{\pi h}{3}[r_1^2 + r_2^2 + r_1 r_2]$$

$$= \frac{22}{7} \times \frac{24}{3}[(7)^2 + (14)^2 + 7 \times 14] \text{ cm}^3$$

$$= \frac{22 \times 8}{7}[49 + 196 + 98] \text{cm}^3$$

$$= \frac{22 \times 8}{7} \times 343 \text{ cm}^3 = 8624 \text{ cm}^3$$

As $1 \text{ cm}^3 = \frac{1}{1000} l$;

$\therefore$ Cost of milk $= ₹ \frac{8624}{1000} \times 25 = ₹ 215.60$

$\therefore$ Area of metal sheet used = Curved surface area of cone + Area of circular base

$$= \pi l(r_1 + r_2) + \pi r_1^2$$

$$= \left[\frac{22}{7} \times 25(7 + 14) + \frac{22}{7} \times (7)^2\right] \text{cm}^2$$

$$= \left[\frac{22}{7} \times 25 \times 21 + \frac{22}{7} \times 7 \times 7\right] \text{cm}^2$$

$$= (1650 + 154) \text{ cm}^2 = 1804 \text{ cm}^2.$$

**Question 102.**

A solid copper sphere of surface area 1386 cm² is melted and drawn into a wire of uniform cross-section. If the length of the wire is 31.5 m, find the diameter of the wire

**Solution:**

Let radius of sphere $= r$ cm

Surface area of sphere $= 1386 \text{ cm}^2$

$\Rightarrow \quad 4\pi r^2 = 1386$

$\Rightarrow \quad r^2 = \frac{1386 \times 7}{4 \times 22} = \frac{63 \times 7}{4}$

$\Rightarrow \quad \therefore \; r = \frac{21}{2}$ cm

Let diameter of wire $= d$ cm

Given, length, $h = 31.5$ m $= 3150$ cm

A.T.Q.,

Volume of copper sphere melted = Volume of wire drawn

$$\frac{4}{3}\pi r^3 = \pi\left(\frac{d}{2}\right)^2 h$$

$\Rightarrow \quad \frac{4}{3}\pi\left(\frac{21}{2}\right)^3 = \pi\left(\frac{d}{2}\right)^2 \times 3150$

$\Rightarrow \quad \frac{4}{3} \times \left(\frac{21}{2}\right)^3 \times \frac{1}{3150} = \left(\frac{d}{2}\right)^2$

$\Rightarrow \quad \left(\frac{d}{2}\right)^2 = \frac{49}{100}$

$\Rightarrow \quad \frac{d}{2} = \frac{7}{10} \Rightarrow d = 1.4$ cm.

$\therefore$ Diameter of uniform cross-section of the wire is 1.4 cm.

**Question 103.**

A solid right circular cone of diameter 14 cm and height 4 cm is melted to form a hollow hemisphere. If the external diameter of the hemisphere is 10 cm, find its internal diameter. Also find the total curved surface area of the hemisphere.

**Solution:**

Let internal diameter of hemisphere = $r$ cm

Given, external diameter of hemisphere = 10 cm, external radius, R = 5 cm

**For cone:** diameter of the base = 14 cm; height = 4 cm

A.T.Q., Volume of cone = Volume of hemisphere

$$\frac{1}{3}\pi r_1^2 h = \frac{2}{3}\pi[R^3 - r^3]$$

$$\frac{1}{3} \times \pi \times (7)^2 \times 4 = \frac{2}{3}\pi[(5)^3 - r^3]$$

$$98 = 125 - r^3$$

$\Rightarrow$ $r^3 = 125 - 98 = 27$

$\Rightarrow$ $r = 3$ cm

$\therefore$ Internal diameter of the hemisphere = 6 cm

Total curved surface area of the hemisphere

$$= [2\pi(5)^2 + 2\pi(3)^2 + \pi\{(5)^2 - (3)^2\}] \text{ cm}^2$$

$$= (50\pi + 18\pi + 16\pi) \text{ cm}^2$$

$$= 84 \times 3.14 \text{ cm}^2 = 263.76 \text{ cm}^2.$$